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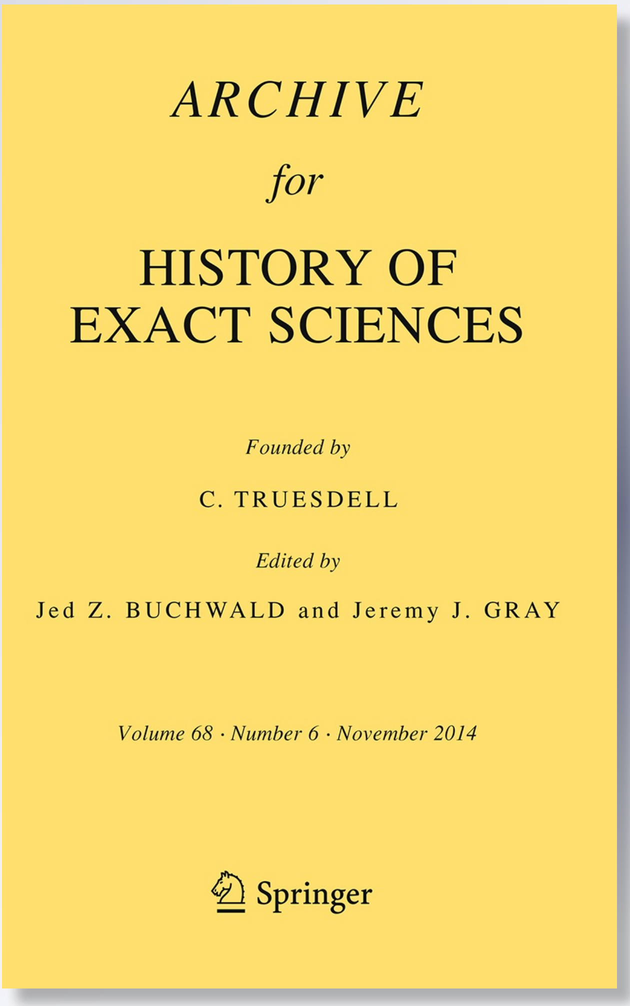
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Abstract The eclipse predictor (or Saros dial) of the Antikythera mechanism provides a wealth of astronomical information and offers practically the only possibility for a close astronomical dating of the mechanism. We apply a series of constraints, in a sort of sieve of Eratosthenes, to sequentially eliminate possibilities for the epoch date. We find that the solar eclipse of month 13 of the Saros dial almost certainly belongs to solar Saros series 44. And the eclipse predictor would work best if the full Moon of month 1 of the Saros dial corresponds to May 12, 205 BCE, with the exeligmos dial set at 0. We also examine some possibilities for the theory that underlies the eclipse times on the Saros dial and find that a Babylonian-style arithmetical scheme employing an equation of center and daily velocities would match the inscribed times of day quite well. Indeed, an arithmetic scheme for the eclipse times matches the evidence somewhat better than does a trigonometric model.

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Some of the main results of this paper were presented in Leiden, on June 20, 2013, at the week-long conference, *The Antikythera Mechanism: Science and Innovation in the Ancient World* organized by Niels Bos, Mike Edmunds, Alexander Jones, Onno van Nijf, and Rien van de Weygaert. A PowerPoint of our presentation, "On the Epoch of the Antikythera Mechanism," was posted on the conference Web site, available to participants (Evans and Carman 2013). A shorter version of the talk was presented at the International Congress of History of Science, Technology and Medicine, in Manchester, UK, on July 22, 2013, under the title "Approaches for the epoch of the Antikythera mechanism," in Session S092-B.

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1 Introduction

Lunar eclipses were predicted with reasonable accuracy by the Babylonians using the Saros cycle, involving a schematic distribution in the arrangement 8-8-7-8-7- (Steele 2000a). Here, each 8 represents a group or sequence of eight eclipses at 6-month intervals. And each 7 represents a sequence of seven eclipses at 6-month intervals. The last eclipse of any 8 or 7 group is separated from the first eclipse of the following group by only a 5-month interval. Thus, the locations of the 5-month intervals are indicated by the hyphens. The total number of lunar eclipses in one cycle is $8+8+7+8+7 = 38$, and the duration of the cycle of 38 lunar eclipses is $47 + 47 + 41 + 47 + 41 = 223$ synodic months. Of course, this is a cycle, so the starting point is arbitrary. Thus, one can also have lunar eclipses in the patterns 8-7-8-7-8- or 7-8-7-8-8-, for example. And in an actual sequence of predicted eclipses, it is by no means necessary to start at the beginning of an 8 or a 7 group. Solar eclipses were also predicted by the Babylonians using the same sort of 8-8-7-8-7- scheme. Perhaps we should refer to these as solar eclipse possibilities (EP), for in Babylonian astronomy there was no reliable way to determine which solar eclipses would actually be visible from Babylon.[1]

The Antikythera mechanism (AM) is a Greek geared astronomical computing machine, built sometime between the late third century and the early first century BCE, which was recovered from an ancient shipwreck in 1901. A remarkable feature of a recent study by Freeth et al. (2008) is a demonstration that eclipses were predicted on the lower back dial of the mechanism by means of the Saros cycle. Only a small portion of the inscriptions on the Saros dial are preserved. Nevertheless, Freeth et al. were able to show that the predictive scheme is consistent with a Babylonian-style 8-8-7-8-7- pattern. The reconstruction is greatly aided by the presence of index letters in the glyphs for the eclipses. Each month box that bears an eclipse glyph is labeled by a Greek letter that shows where the glyph stands in the sequence. Thus, even though a large chunk of the dial is missing, it is possible to be sure exactly how many eclipse glyphs would have been carried by most of the missing part of the dial. Freeth et al. attempted a fit of the eclipses inscribed on the Antikythera mechanism, in an effort to determine which year or years might best correspond to the first year of the Saros dial. This would be one important approach, among several, to dating the mechanism. Other methods include archeological dating of the ship's other cargo, radiocarbon dating of the ship's timbers, and the dating of the Greek inscriptions on the mechanism by means of the forms of the letters. Of course, the first two of these do not directly give any information about the mechanism itself, while the third (dating by letter forms) has an uncertainty of perhaps as much as plus or minus a century. The Saros dial is practically the only *astronomical* feature of the mechanism that could possibly afford a more tightly constrained dating.

[1] The Babylonians had no way to treat the lunar parallax. Some writers characterize also the lunar eclipses in a Saros scheme as "eclipse possibilities." The advantage of this usage is that it acknowledges that the Babylonian astronomers knew that sometimes a predicted lunar eclipse does not occur or that sometimes one occurs in the wrong month. Still, the prediction of lunar eclipses was pretty accurate, so to describe these predictions as giving mere possibilities seems an unduly positivistic characterization of what the Babylonians were up to.

In their analysis of the mechanism's Saros dial, Freeth et al. fitted the extant eclipse glyphs to modern computed eclipse data. A great deal of uncertainty is involved in deciding what to do about penumbral eclipses, easily predicted by modern theory, but rarely if ever mentioned by ancient astronomers. If they did not allow any penumbral eclipses to be counted as corresponding to glyphs on the mechanism, Freeth et al. found that they had no possible epoch dates. If, on the other hand, they allowed themselves the flexibility of counting an occasional penumbral eclipse as one of the Babylonian eclipse possibilities, they found they had over a hundred possible fits for epoch date. Clearly, one would like a result in between these two outcomes.

In this paper we shall offer a fresh analysis of the eclipses predicted on the Saros dial of the Antikythera mechanism, with the goal of establishing the epoch year for this feature of the mechanism. By epoch year we mean the starting year or "year 1" from which the ancient mechanic reckoned time for the purpose of the mechanism's display. The epoch is not necessarily the same as the date of manufacture, though it would be surprising if they were very widely separated in time.

Our working hypothesis is that the Antikythera mechanism's eclipse glyphs should be fitted, not to modern computations of eclipses, but to Babylonian practice. Such an analysis has the advantage of removing the spurious penumbral eclipses from consideration. Moreover, the 8-8-7-8-7- scheme is far from perfect: it can predict an eclipse that does not actually occur, fail to predict an eclipse that does occur, or predict an eclipse for the wrong month. Thus, it is a fallacy to suppose that the epoch of the Antikythera mechanism's Saros dial could be reliably determined simply by matching its predictions against real eclipses. Although it is a priori likely that the Babylonian Saros scheme was the ultimate source for the predictive scheme represented on the mechanism, one should not simply assume this without scrutiny. We shall present evidence below that the eclipse prediction scheme on the Antikythera mechanism really does derive from the Babylonian example.

Of course, in parts of our analysis, we shall make considerable use of theoretically computed phenomena, such as nodal distances and eclipse times. We will show that it is possible to introduce constraints sequentially, as with a sort of "sieve of Eratosthenes," to systematically restrict the number of possible solutions. The tools at our disposal include as follows: the distributions of the solar and lunar eclipse glyphs on the Antikythera mechanism, which may be fitted to Babylonian patterns in a number of different ways; the connection between the solar and lunar patterns, which may be compared with similar connections in the Babylonian material; the "omitted solar eclipses" (defined below), which may be used to classify the solar eclipses and to establish their places in the 8-8-7-8-7- sequence without ambiguity; nodal elongations, which may be used to narrow the range of fits; eclipse times, which may be used to restrict the solutions to a single Saros eclipse series, and even to rule out two fits of every three in that series. When these approaches are used in conjunction, it is possible to arrive at a single most likely epoch for the eclipse predictor.

2 Terminology

We use "8-" to stand for the run of 47 synodic months beginning with the month in which occurs the first eclipse of an 8-eclipse group. In the following pictorial representation of an 8-, each E_i denotes a month with a (say, lunar) eclipse and each x denotes a month without an eclipse:

$$E_1xxxxxE_2xxxxxE_3xxxxxE_4xxxxxE_5xxxxxE_6xxxxxE_7xxxxxE_8xxxx.$$

Sometimes, it is convenient to talk about time intervals between eclipses: We shall use "interval" to mean a time difference. Thus, from lunar eclipse E_1 to lunar eclipse E_2 is an interval of six synodic months. From E_8 to the first eclipse of the following group is an interval of 5 months. On the other hand, it is often convenient to count actual months, and we shall use "gap" to indicate a run of empty, eclipseless months. Thus, the gap between the months containing E_1 and E_2 consists of five months, while the gap between the months containing E_8 and the following eclipse consists of 4 months.

"8" stands for the group of 8 eclipses, but not counting the four empty months at the end. Thus, an 8 begins with an eclipse and ends with an eclipse. An 8 is 43 months long:

ExxxxxExxxxxExxxxxExxxxxExxxxxExxxxxExxxxxE.

"7-" represents the interval from the first eclipse of a 7-eclipse group though the four empty months after the last eclipse of the group. A 7- is 41 months long:

ExxxxxExxxxxExxxxxExxxxxExxxxxExxxxxExxxx.

"7" is the same, but without the four empty months at the end. A 7 is 37 months long:

ExxxxxExxxxxExxxxxExxxxxExxxxxExxxxxE.

A number of different concepts involve the term "Saros," so we should take a moment to make their meanings clear. Here, we follow the definitions of Steele (2000a, p. 424). By "Saros cycle," we mean the period of 223 synodic months containing 38 eclipse possibilities. A "Saros scheme" will mean a particular distribution of eclipse possibilities within a Saros cycle. Different Saros schemes were used by the Babylonian astronomers, as the 8-8-7-8-7- distribution of eclipses needed to be re-calibrated from time to time, to maintain fidelity to the phenomena. By "Saros series," we refer to a collection of eclipse possibilities, each separated by one Saros cycle of 223 synodic months from the preceding eclipse possibility. If a solar eclipse takes place this month, then another, with similar characteristics, will occur 223 months from now. These two eclipses are part of a single series, which may be assigned a standard number. One speaks of solar Saros series 46, for example.

A single Saros series may persist for 1,200 years or more. At any moment in history, about 40 Saros series are in progress. Each eclipse can be labeled by its Saros series number in the standard catalogs of Espenak (Five Millennium Catalog of Solar Eclipses; Five Millennium Catalog of Lunar Eclipses). Saros series of solar eclipses are given even numbers if they occur near the descending node of the Moon's orbit and odd numbers if they occur near the ascending node. When a new solar Saros series begins at the descending node, the new Moon occurs about 18° east of the descending node (and so below the ecliptic). A partial eclipse of the Sun will be visible from southern parts of the Earth—just the northern limb of the Sun will be grazed. At progressively later eclipses in the same series, the Sun will be centrally eclipsed, and then, gradually this Saros series will disappear off the south limb of the Sun (Espenak, Eclipses, and the Saros). Lunar Saros series have odd numbers if the eclipses occur with the Moon near the descending node of its orbit and even numbers if they occur with the Moon near the ascending node.

The reason for the finite lifetime of a Saros series is that the Saros relation 223 synodic months (sm) = 242 draconic months (dm) is not exact. A more precise correspondence is 223 sm = 241.99868 dm (Smart 1977, p. 420). Thus, when 223 synodic months have elapsed, the time elapsed in draconic months is <242 by 0.00132 dm, which corresponds to 0.475° of mean motion with respect to the node. That is, from one eclipse to the next in the same Saros series, the position of the mean conjunction moves westward by about 0.475°. Now, the Sun's ecliptic limits from the node for a solar eclipse to be possible vary from a maximum of about 18.4° to a minimum of about 15.4° (Smart 1977, p. 390). Let us take 17° as a round value. The width of the nodal zone is thus 34°; the position of the mean conjunction recedes by about 0.475° per Saros and so takes roughly 72 Saros cycles to cross the whole nodal zone, which amounts to about 1,300 years. If about 40 Saros series are in progress, on the average, at any one time, then a Saros series must end (and another must begin) roughly every 1,300/40 = 33 years or so. This is one reason why the Babylonian astronomers needed to re-calibrate their Saros eclipse scheme from time to time.

3 The lunar eclipse pattern on the Antikythera mechanism

The 14 lunar eclipses attested on the surviving portion of the dial are in month cells 20, 26, 61, 67, 79, 114, 120, 125, 131, 137, 172, 178, 184, and 190.[2] These are the data that a Babylonian 8-8-7-8-7- eclipse distribution must match. Freeth et al. argued that only a single possible reconstruction was consistent with the preserved lunar eclipse glyphs and index letters. Actually, there are two possible lunar eclipse patterns, which

[2] Original publication of the eclipse glyphs is then known: (Freeth et al. 2008). These (both the journal article and the online Supplementary Notes) contained some errors in the presentation of the eclipse data. These have been corrected in a new version of the Supplementary Notes posted at http://www.nature.com/nature/journal/v454/n7204/extref/nature07130-s1.pdf, to which readers should now refer. (Most of these errors were corrected by Beatriz Bandeira of the Universidad Nacional Tres de Febrero of Argentina). We have taken the times of the eclipses from the most recent analysis made by Alexander Jones in a still unpublished paper. They differ in some cases from the times published in the Supplementary Notes of Freeth et al. (2008). The following differences should be noted. We list first the eclipse cell and eclipse type, then the reading of Freeth et al., and finally Jones's reading. 13S: day 1 or 4, day 1. 78S: night 1, day 1. 120M: day 8, day 6. 172M: no reading, night 6. 172S: no reading, day 12. 178S: most unclear, day 9. In all cases, we followed Jones's readings. We excluded from the time analysis three eclipses: 120M because Jones's reading is uncertain and 67S and 72H because the reading is incomplete. For S119 both Jones and Feeth et al. 2008 read ι[ ] which could be 10, 11 or 12, so we assume the mean value, 11. A lunar eclipse in month 61 has recently been discovered (Anastasiou et al. 2014). Our eclipse analysis was completed before this discovery was made; but the new glyph fit right into the already reconstructed Babylonian pattern with no changes required.

are displayed in Table 1. The lunar pattern that we call α was identified by Freeth et al. The other possible solution we will call lunar pattern β. (See "Appendix 1" for an explanation).

In the first column of Table 1, the eclipse possibilities for one Saros period are labeled from 1 to 38. The next two columns illustrate the features of lunar pattern α. The month in which each EP falls is listed in column two, the months being numbered from the first cell of the Antikythera mechanism's Saros dial. Cell numbers in bold type indicate surviving lunar eclipse inscriptions. The cell numbers written in parentheses are the other eclipse possibilities predicted by pattern α. The third column shows the grouping of eclipses into a 7-8-7-8-8- pattern. Note that, in this case, the first 7- group is not complete, as only six of the eclipses appear at the beginning of the chart. Alternatively, one may think of this 7- group as beginning with an eclipse at month 219 of the previous Saros cycle. The 5-month intervals are indicated by the heavy bars. The beginning of the double 8- group is indicated by the doubled heavy bar.

Lunar pattern β (in the pattern 8-8-7-8-7-) is displayed in the final two columns. Note that the only difference between these two solutions occurs in the 37th eclipse possibility: In pattern α, this eclipse falls in month 214, while in pattern β it falls in month 213. The cells for eclipse possibility 37 are shaded to indicate that the month number of this eclipse differs between the two patterns. As there are no visible glyphs beyond month 190, both possibilities are consistent with the evidence. We shall keep both options open for the time being.

Conclusion of Sect. 3: Two patterns, α and β, are consistent with the preserved lunar eclipse glyphs and with a Babylonian-style 8-8-7-8-7- eclipse scheme.

4 The solar eclipse pattern on the Antikythera mechanism

The Babylonians used a similar 8-8-7-8-7- distribution to obtain possibilities for solar eclipses. On the Saros dial of the Antikythera mechanism, solar EPs are inscribed in months 13, 25, 72, 78, 119, 125, 131, 137, 172, 178, and 184. Freeth et al. identified one 8-8-7-8-7- distribution scheme of solar eclipses consistent with the extant glyphs. But there are actually seven different 8-8-7-8-7- schemes consistent with the extant glyphs. These are illustrated in Table 2. (See "Appendix 2" for an explanation of how we arrive at these solutions). As before, the extant eclipse glyphs are indicated in bold type. The 5-month intervals (4-month gaps) for each solution are indicated by the heavy bars. The beginning of the double 8- group is indicated by the double bar. In Table 2, the solar eclipses that can come in a variable month (dependent on which of the seven patterns is actually represented on the Antikythera mechanism) are shaded. The un-shaded eclipses would occur in the same months, no matter which of the seven patterns was actually used. Taking the solar and lunar solutions together, there are therefore 14 solutions for placement of the eclipses on the Saros dial that are consistent with the preserved inscriptions and with Babylonian-style 8-8-7-8-7- schemes, which we call 1α, 2α, 3α, 4α, 5α, 6α, 7α, 1β, 2β, 3β, 4β, 5β, 6β, and 7β.

Table 1 Lunar eclipse patterns consistent with the extant glyphs on the Antikythera mechanism and with an 8-8-7-8-7-Babylonian-style scheme

	Lunar Pattern α		Lunar Pattern β	
Eclipse number	**Month of eclipse**			**Month of Eclipse**
1	(2)			(2)
2	(8)			(8)
3	(14)	7	8	(14)
4	**20**			**20**
5	**26**			**26**
6	(32)			(32)
7	(37)			(37)
8	(43)			(43)
9	(49)			(49)
10	(55)	8	8	(55)
11	**61**			**61**
12	**67**			**67**
13	(73)			(73)
14	**79**			**79**
15	(84)			(84)
16	(90)			(90)
17	(96)			(96)
18	(102	7	7	(102)
19	(108)			(108)
20	**114**			**114**
21	**120**			**120**
22	**125**			**125**
23	**131**			**131**
24	**137**			**137**
25	(143)	8	8	(143)
26	(149)			(149)
27	(155)			(155)
28	(161)			(161)
29	(167)			(167)
30	**172**			**172**
31	**178**			**178**
32	**184**			**184**
33	**190**		7	**190**
34	(196)	8		(196)
35	(202)			(202)
36	(208)			(208)
37	(214)			(213)
38	(219)			(219)

Table 2 Solar eclipse patterns consistent with the extant glyphs on the Antikythera mechanism and with an 8-8-7-8-7- Babylonian-style scheme

Eclipse Possibility	Solar Case i			Solar Case ii		Solar Case iii	
	1	2	3	4	5	6	7
1	(2)	(2)	(2)	(2)	(2)	(2)	(2)
2	(8)	(8)	(8)	(7)	(7)	(8)	(8)
3	**13**	**13**	**13**	**13**	**13**	**13**	**13**
4	(19)	(19)	(19)	(19)	(19)	(19)	(19)
5	**25**	**25**	**25**	**25**	**25**	**25**	**25**
6	(31)	(31)	(31)	(31)	(31)	(31)	(31)
7	(37)	(37)	(37)	(37)	(37)	(37)	(37)
8	(43)	(43)	(43)	(43)	(43)	(43)	(43)
9	(49)	(49)	(49)	(49)	(49)	(49)	(49)
10	(54)	(55)	(54)	(54)	(54)	(55)	(55)
11	(60)	(60)	(60)	(60)	(60)	(60)	(60)
12	(66)	(66)	(66)	(66)	(66)	(66)	(66)
13	**72**	**72**	**72**	**72**	**72**	**72**	**72**
14	**78**	**78**	**78**	**78**	**78**	**78**	**78**
15	(84)	(84)	(84)	(84)	(84)	(84)	(84)
16	(90)	(90)	(90)	(90)	(90)	(90)	(90)
17	(96)	(96)	(96)	(95)	(96)	(96)	(96)
18	(101)	(101)	(101)	(101)	(101)	(101)	(102)
19	(107)	(107)	(107)	(107)	(107)	(107)	(107)
20	(113)	(113)	(113)	(113)	(113)	(113)	(113)
21	**119**	**119**	**119**	**119**	**119**	**119**	**119**
22	**125**	**125**	**125**	**125**	**125**	**125**	**125**
23	**131**	**131**	**131**	**131**	**131**	**131**	**131**
24	**137**	**137**	**137**	**137**	**137**	**137**	**137**
25	(142)	(143)	(143)	(142)	(142)	(143)	(143)
26	(148)	(148)	(148)	(148)	(148)	(148)	(148)
27	(154)	(154)	(154)	(154)	(154)	(154)	(154)
28	(160)	(160)	(160)	(160)	(160)	(160)	(160)
29	(166)	(166)	(166)	(166)	(166)	(166)	(166)
30	**172**	**172**	**172**	**172**	**172**	**172**	**172**
31	**178**	**178**	**178**	**178**	**178**	**178**	**178**
32	**184**	**184**	**184**	**184**	**184**	**184**	**184**
33	(189)	(189)	(189)	(189)	(189)	(190)	(190)
34	(195)	(195)	(195)	(195)	(195)	(195)	(195)
35	(201)	(201)	(201)	(201)	(201)	(201)	(201)
36	(207)	(207)	(207)	(207)	(207)	(207)	(207)
37	(213)	(213)	(213)	(213)	(213)	(213)	(213)
38	(219)	(219)	(219)	(219)	(219)	(219)	(219)

Conclusion of Sect. 4: Seven patterns, 1–7, are consistent with the preserved solar eclipse glyphs and with a Babylonian-style 8-8-7-8-7- eclipse scheme. There are therefore 14 ways to reconstruct the Saros dial.

5 Britton, Steele, and alternative Steele rules for linking solar and lunar patterns

Babylonian practice favored linking a solar eclipse pattern to a lunar eclipse pattern, which could perhaps allow us to discard some solar patterns or, at least, to prefer the others. These links have been discussed by Britton and by Steele, with somewhat different results.

Britton (1989, pp. 21–24) described the relationship between lunar and solar eclipses in these terms: When the Moon pattern is 8-8-7-8-7, the Sun pattern is 7-8-7-8-8 (exactly the reverse) but starts three eclipse possibilities earlier.[3] So, if we are given one lunar eclipse pattern, we automatically have the corresponding solar pattern. We will call this the *Britton rule*.

But Steele (2000a, p. 443) has proposed another rule: "There is in fact an even closer link between the lunar and solar Saroi, at least in the period after −250. From here until at least −70, it would seem that the solar and lunar Saroi have the same 8-7-8-7-8 distribution, with the solar Saros always starting 4 eclipse possibilities earlier than the lunar Saros." So, according to Steele, when the Moon pattern is 8-7-8-7-8, the Sun pattern is the same, 8-7-8-7-8, but starts 4 EP earlier. We will call this the *Steele rule*.

According to Steele, the differing results may be accounted for by the different input information: "Britton's rule is derived from a theoretical model of nodal elongation (proposed by Aaboe) and (I think) agrees with what we find on the so-called Saros Canon texts, which are a small group of theoretical texts from Babylon. The rule I gave is based upon actual preserved records of eclipse predictions and observations found in Babylonian texts. Thus, Britton's rule is a theoretical rule (reflecting also a particular Babylonian theory), whereas my rule is a reflection of what the Babylonians did in practice" (John Steele, personal communication).

In practice, the Babylonians re-calibrated their Saros schemes from time to time, when they got too far out of synchrony with the eclipse phenomena. Re-calibration involves a discontinuity in the run of repeating 8-8-7-8-7- patterns (the details will be discussed below). Once a re-calibration had occurred, the scheme typically could run for 2–5 Saros cycles without readjustment. Often, the Babylonian re-calibration occurred a few Saros cycles later than we might judge to be ideal, which means that some time had to elapse before the Babylonian astronomers perceived that the errors in the eclipse predictions had built up sufficiently so that re-calibration was required. As we shall see, the successive Babylonian re-calibrations of the Saros provide a useful dating tool.

3 This is also discussed in Aaboe et al. (1991, p. 17).

On rare occasions, it is possible that the scheme for solar eclipses was re-calibrated, but the scheme for lunar eclipses was left unchanged for some time.[4] In such instances, what we shall call the *alternative Steele rule* would be in force, as it is effectively a different form of the Steele rule. When the alternative Steele rule is in effect, the solar and lunar Saros schemes have the same 8-7-8-7-8- distribution as one another, but with the solar scheme always starting 4 eclipse possibilities *later* than the lunar scheme. We do not know whether this was a genuine rule of Babylonian eclipse theory—it could have resulted simply from correcting the solar calibration but leaving the lunar calibration unchanged for a time. Historically, the alternative Steele rule seems to have been in effect from about -71 at least to the end of the first century BCE. It may also have happened that the alternative Steele rule was in effect from time to time for shorter stretches, as we do not know exactly when all the recalibrations were performed.[5]

Babylonian tabulations of lunar and solar eclipses are typically found on different tablets—it is not common to find them in one document. Therefore, a Greek astronomer could also unwittingly arrive at the alternative Steele rule if he made use of a canon of solar eclipses and a canon of lunar eclipses that ran for an overlapping sequence of years, if the solar canon represented a post-recalibration sequence and the lunar canon represented a pre-recalibration sequence. We might expect it to be more likely that such a circumstance would obtain shortly after a Babylonian recalibration.

As we have seen, two lunar eclipse schemes, α and β, are consistent with the extant lunar eclipse glyphs; and seven solar eclipse schemes, numbered 1 through 7, are consistent with the extant solar eclipse glyphs. Thus, we have $2 \times 7 = 14$ possibilities for the combined solution. However, if the eclipse scheme on the Antikythera mechanism's Saros dial incorporates one of the possible Babylonian links between the solar and lunar patterns, there may be fewer viable alternatives. It is an easy matter to examine each of the 14 possible combinations to determine which, if any, is consistent with the Britton, Steele, or alternative Steele rules. We obtain the following results:

Britton rule If Lunar Pattern α and Solar Pattern 7 are represented on the Antikythera mechanism, these would be connected by the Britton rule.
If Lunar Pattern β and Solar Pattern 6 are represented on the mechanism, these would be connected by the Britton rule.
Steele rule If Lunar Pattern α and Solar Pattern 6 are represented on the mechanism, these would be connected by the Steele rule.
If Lunar Pattern β and Solar Pattern 2 are represented on the mechanism, these would be connected by the Steele rule.

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[4] That is, one modifies the solar Saros scheme without modifying the lunar scheme, in such a way that the solar scheme that began 4 EP earlier than the lunar one now begins 4 EP later than the lunar one. This could sound odd if one supposes that the solar schemes originated as modifications of the lunar schemes. Why would you recalibrate the solar and not the lunar if the solar is based on the lunar? But it seems that in the case of re-calibration the dependence is in the other direction. Steele (2000a, p. 443) suggests that one particular lunar recalibration, in -110, was unnecessary but nevertheless took place to coincide with a solar re-calibration. He also suggests that "Since solar eclipses can occur at greater nodal elongations than lunar eclipses, it was necessary to revise the solar Saros scheme more often than the lunar Saros."

[5] For example, Steele (2000a, p. 442) points out that the circumstances of the -110 recalibration are not completely clear.

Alternative Steele rule If Lunar Pattern β and Solar Pattern 7 are represented on the mechanism, these would be connected by the alternative Steele rule.

So, instead of 14 combinations, we have only five (2β, 6α, 6β, 7α, and 7β) that should perhaps be preferred. In particular, none of solar patterns 1, 3, 4, 5 is consistent with any version of the Babylonian linking convention.

We might expect the Antikythera mechanism to reflect the Steele or alternative Steele rules, as these are deduced from what the Babylonian astronomers did in practice, rather than the Britton rule, which reflects a particular set of Babylonian theoretical texts. In particular, we might expect the alternative Steele rule to show up if the mechanism was designed after −71, or if it was designed at some earlier date but shortly after a Babylonian re-calibration of the solar Saros. However, at this stage, we leave all possibilities open—Steele, alternative Steele, or Britton rule, or something else entirely.

Conclusion of Sect. 5: Babylonian practice favored linking a solar eclipse scheme to a lunar eclipse scheme. Steele and Britton have published different versions of these rules. The Steele rule or the alternative Steele rule should be preferred since they reflect actual Babylonian eclipse records for the period covering the likely date of manufacture of the AM. Some, but not all, of the 14 solutions mentioned in Sect. 4 are consistent with the Steele or with the Britton rule. However, we do not use any of the linking rules to eliminate solutions.

6 Node conventions: a basic link between solar and lunar patterns

A key historical fact is that in the Babylonian records stretching over hundreds of years, a solar eclipse that is considered to begin the 8-8- double group is generally an eclipse at the descending node of the Moon's orbit. To confirm this, we have computed the argument of the latitude of the Moon (Moon's mean distance from the ascending node)[6] at each eclipse beginning a solar double 8- in the Babylonian eclipse record published by Steele. For actually occurring eclipses, we input the time of the midpoint of the eclipse, taken from the Espenak catalog. For eclipses predicted in the Babylonian Saros scheme but not actually occurring, we used the time of conjunction. This analysis confirms that we are dealing with a reliable Babylonian convention, carefully preserved across multiple recalibrations. The Saros cycle contains an even number (38) of solar EPs, so placing one Saros cycle after another does not disrupt the regular alternation between the ascending and descending nodes. As we shall see below, when re-calibrations of the Saros were required, this was usually done by inserting a 47-month cycle (with 8 solar EPs) between the ending of one Saros cycle and the beginning of another. Such a recalibration also did not disrupt the regular alternation between the nodes.

[6] For the Moon's argument of latitude (the mean angular distance of the Moon from its ascending node), see (Meeus 2009, p. 338, Eq. 47.5). In our calculations, we used an Excel program, based on Meeus's formula, written by Dennis Duke.

An even simpler way to see the pattern is the following. It is somewhat easier to work with the first eclipse of the *second* 8- in a 8-8- double group. (This is because the Saros series involving the first eclipse of the first 8- in the double 8- often ended, but was continued for some time by the Babylonians, before they realized that a re-calibration was in order. Hence, a number of these predicted eclipses did not actually occur, so their Saros series numbers are undefined). Here, we give samples of solar eclipses, each of which is a first eclipse of the second 8- in a 8-8- in Steele's reconstruction of the Babylonian eclipse records. These are accompanied by their solar Saros series number:[7]

Some solar eclipses, first in the second 8- of a 8-8-

Date	Solar Saros Series
−351 May 3	42
−333 May 14	42
−239 Apr 25	44
−221 May 6	44
−181 Mar 15	46
−163 Mar 26	46
−105 Feb 14	48
−87 Feb 25	48
−29 Jan 15	50
−11 Jan 26	50

As we can see, the solar eclipse beginning the second 8- of an 8-8- always belongs to an even-numbered solar Saros series—and is therefore an eclipse at the descending node. The same is therefore true of the first eclipse of the first 8- of the double 8-. During the course of a recalibration, it is not always easy to decide where to put the divisions between the 8- groups and the 7- groups. This is because the eclipse records are spotty: Only isolated eclipse dates are actually preserved and only in a limited number of cases do we have an explicit statement that a particular eclipse was preceded by a 5-month interval. But whenever the Babylonian 8-8-7-8-7- scheme is running regularly, the pattern seems to hold, that the second 8- of a double 8- begins with a solar eclipse at the descending node—and this over the whole period that is possible for the construction of the Antikythera mechanism.

A similar examination of Steele's list of Babylonian lunar eclipses shows that in the case of lunar eclipses, *but only after about* −250, the first eclipse of a double-8 group always occurs with the Moon at the ascending node of its orbit. Around −260, a major recalibration occurred, in which the first lunar eclipse of an 8-8- moved from odd lunar Saros series to even ones. This was apparently connected with adoption of the Steele rule, which, as we saw above, went into effect around −250. From the middle of the third century BCE down to the end of Steele's table at the beginning of the first century CE (and so for the whole period in which the Antikythera mechanism might have been constructed), the first lunar eclipse of an 8-8- remained at the ascending node.

[7] The groupings of eclipses into 8-8-7-8-7s are from Steele (2009, Table 4). The Saros series numbers for these eclipses come from Espenak (Five Millennium Catalog of Solar Eclipses).

Moreover, this is a logical consequence of the Steele rule. Suppose that there were a cycle in which a solar eclipse fell at the start of a double-8 and that there was also a lunar eclipse this same month. Because it is the start of a solar double-8, we know the Moon was at the descending node at the solar eclipse; therefore, the Moon was at the ascending node at the lunar eclipse of the same month. But the lunar double-8 would start 4 EPs later or earlier (in the Steele or alternative Steele rules, respectively). 4 is an even number, so the Moon must also be at the ascending node that starts the lunar double-8.

Again, here are some examples from Steele's reconstruction—selected lunar eclipses that start the *second* 8- of a 8-8-group:

Some lunar eclipses, first in the second 8- of a 8-8-

Date	Lunar Saros Series
−255 Mar 8	38
−237 Mar 20	38
−197 Jan 27	40
−179 Feb 7	40
−85 Jan 20	42
−67 Jan 30	42

Lunar Saros series have even numbers if they occur with the Moon near its ascending node. Thus, the lunar eclipses beginning the second 8- of an 8-8- have the Moon at the ascending node. The same is true of the lunar eclipses beginning the first 8-. Again, in the course of the recalibrations, things can be unclear. But when the system is running normally, each lunar 8-8- starts with an eclipse in which the Moon is at its ascending node.

Now, if we assume that the eclipse predictor on the AM represents Babylonian practice, then at the first eclipse of the solar double 8- the Moon must be at its descending node and at the first eclipse of the lunar double 8- the Moon must be at its ascending node. We may thin out the 14 solutions by imposing this requirement. In Table 3, for each of the seven solar solutions, we require the first eclipse of the double-8 (the eclipse immediately following the heavy double line) to have the Moon at the descending node D. For example, solar pattern 5 has the first eclipse of the 8-8- in month 7, and this is marked D. Then, alternating D and A, we may easily fill out the rest of the column. For lunar pattern α, the 8-8- begins with month 125, and this is marked A: The Moon must be at its ascending node at the time of this lunar eclipse. Again, the rest of the pattern may be filled put by alternating A, D, A, D, ….

Key are the months that have both solar and lunar eclipses: These are months 125, 131, 137, 172, 178, and 184. The requirements of the Babylonian conventions are highlighted in these months. Then, we may easily cross solar patterns against lunar patterns to see which combinations are permissible. We will use month 125 as our example, but any one of these 6 months would suffice. In month 125, solar pattern 7 requires the Moon to be at A for the solar eclipse; and lunar pattern β requires the Moon to be at D for the lunar eclipse of the same month. These are compatible requirements, since, in the 2 weeks from the lunar to the solar eclipse, the Moon will move to the opposite node. Thus, combination 7β is permissible.

Table 3 Use of the Babylonian node conventions to exclude solutions 1α, 2α, 3β, 4α, 5β, 6β, and 7α. In a month having both a solar and a lunar eclipse, if the Moon is at D for the solar eclipse, it must be at A for the lunar eclipse (or vice versa)

Solar Patterns							Lunar Patterns	
1	2	3	4	5	6	7	α	β
(2)	(2)	(2)	(2)	(2)	(2)	(2)	(2)	(2)
				(7) **D**				
				A		**13 D**		
				D		**A**		
				A		**D**		
				.		**A**		
				.		.		
				.		.		
						.		
		(54) **A**						
			(95) **D**					
					(101)**D**			
125 A	**125 A**	**125 D**	**125 A**	**125 D**	**125 D**	**125 A**	**125 A**	**125 D**
131 D	**131 D**	**131 A**	**131 D**	**131 A**	**131 A**	**131 D**	**131 D**	**131 A**
137 A	**137 A**	**137 D**	**137 A**	**137 D**	**137 D**	**137 A**	**137 A**	**137 D**
(142)**D**								
172 A	**172 A**	**172 D**	**172 A**	**172 D**	**172 D**	**172 A**	**172 A**	**172 D**
178 D	**178 D**	**178 A**	**178 D**	**178 A**	**178 A**	**178 D**	**178 D**	**178 A**
184 A	**184 A**	**184 D**	**184 A**	**184 D**	**184 D**	**184 A**	**184 A**	**184 D**
	(189)**D**							
								(213) **A**

By contrast, combination 1α is ruled out: Solar pattern 1 requires the Moon to be at A for the solar eclipse of month 125, while lunar pattern α requires the Moon also to be at A for the lunar eclipse of the same month. These are incompatible requirements.

By such examinations, it is easy to see that combinations 1α, 2α, 3β, 4α, 5β, 6β, and 7α are all eliminated. Combinations 1β, 2β, 3α, 4β, 5α, 6α, and 7β are still possible.

We note that the Britton rule combinations 6β and 7α are excluded. The Britton rule is not compatible with the Babylonian convention, attested in the Saros record from about −250 onward, of starting a solar 8-8- with the Moon at D for the solar eclipse and starting a lunar 8-8- with the Moon at A for the lunar eclipse.[8] The node conventions, while less restrictive than the Steele rule, are compatible with seven possible solutions, including two (2β and 6α) consistent with the Steele rule and one (7β) consistent with the alternative Steele rule.

Moreover, it is possible to prove that the node conventions are necessarily linked to the Steele and Britton rules. Having the 8-8- of the solar eclipses start at D and the 8-8- of the lunar eclipses start at A automatically entails either the Steele or alternative Steele rules. But having the 8-8- of the solar eclipses start at D and the 8-8- of the lunar eclipses also start at D automatically results in the Britton rule. This is demonstrated in "Appendix 6."

Conclusion of Sect. 6: In the Babylonian eclipse records, over the whole period that is conceivable for the manufacture of the Antikythera mechanism, the solar eclipse that begins the 8-8- (in the 8-8-7-8-7- pattern for the solar eclipses) came at the descending node D. And the lunar eclipse that began the 8-8- of the lunar eclipse pattern came with the Moon at the ascending node A. Invoking this condition eliminates half of the 14 solutions mentioned in Sect. 4. Moreover, the Britton rule is eliminated.

7 The omitted solar eclipses as a tool for the classification of solar eclipses

In a Babylonian Saros scheme, equal numbers of solar and lunar eclipse possibilities are generated—38 each in a 223-month cycle. However, on the Antikythera mechanism, fewer solar eclipses were included than lunar eclipses. Freeth et al. (2008) argued convincingly that this is connected with the idea that if a new Moon occurred too far south of the node of the Moon's orbit, the ancient astronomer may not have considered the eclipse to be visible. Although this practice has not yet been found in the Babylonian material, a similar idea is mentioned in Ptolemy's *Almagest* (vi, 5), where it is connected with lunar parallax effects. According to Ptolemy, a lunar eclipse will generally occur if the mean Moon, when in opposition to the Sun, lies within $15°12'$ of the node (Toomer 1984, p. 287). However, the cutoff criterion for solar eclipses is asymmetric: A solar eclipse will generally appear for the northern

[8] However, the Britton rule could have been compatible with Babylonian practice before about −260: As we have seen, during the earlier period, while a solar 8-8- began with the Moon at D for the solar eclipse, the lunar 8-8- also began with the Moon at D for the lunar eclipse.

inhabited latitudes of the Earth if at the time of conjunction the center of the true Moon is no more than 17°41′ north of the node, or no more than 8°22′ south of the node, measured along the Moon's inclined path (Toomer 1984, p. 286). (Ptolemy goes on to add that the cutoffs for the center of the mean Moon are: 20°41′ to the north of the node or 11°22′ to the south). Although the *Almagest* is several centuries later than the Antikythera mechanism, it is not unreasonable to suppose that similar rules governing solar eclipses were in place earlier. Freeth et al. showed that, for rather narrow ranges of the three cutoff parameters (limit of lunar, limit of northern solar, and limit of southern solar eclipses), the predictions of their generating scheme (based on the mean motions of the Saros cycle plus an asymmetric criterion for elongation from the node) can indeed match the evidence. These authors also remarked that "it does not appear to be possible to generate the Antikythera scheme [of solar eclipses] by a simple pattern of excisions from one of the Babylonian schemes" (Freeth et al. 2008, Supplementary Notes, p. 36). Finally, Freeth et al. experimented with schemes that incorporate the first anomalies of the Sun's and Moon's motion; but these did not lead to any real improvement in the simpler scheme based on mean motion and asymmetric cutoffs.

However, we suggest that since the original EPs simply follow the schematic 8-8-7-8-7- pattern, it may be questioned whether an elaborate month-by-month calculation of nodal elongation (whether based on mean or true positions) lies behind the pattern of excluded solar eclipses. This would have made the suppression of occasional solar eclipses far more work than the original listing of eclipse possibilities for the Sun and Moon. We shall show that a simple system for omission of unwanted solar eclipses is probably possible. The basic rule is: at each node, omit the southernmost eclipse in each "diagonal sequence." (This term will be defined below).

In Fig. 1, we display the positions of the solar eclipse glyphs from the Saros dial—but arrange them instead on a replica of the Metonic calendar spiral. The synodic months are numbered here and there for convenience. Parallelograms mark the solar eclipse possibilities that are actually preserved on the mechanism. We place the eclipse symbols consecutively, with the first preserved solar EP falling at month 13, as on the mechanism's Saros dial. As we have just seen, seven different reconstructions are consistent with the extant solar glyphs and the Babylonian node conventions: In Fig. 1, we follow solar pattern 7. Patterns 1–6 would give slightly different diagrams, though the positions of the parallelograms would be the same in all. The circles in Fig. 1 mark the locations of solar eclipse possibilities that are called for by pattern 7, but that are in fact missing from the Saros dial. The double bar in Fig. 1 indicates the beginning of the 8-8- in solar pattern 7; therefore, the Moon must be at D for the first solar eclipse following this double bar (month 13). Let us start reckoning from this moment in the Saros cycle. This eclipse is therefore labeled D_1, and all the remaining eclipses may be labeled D and A alternately. The tendency of eclipses to radial grouping is striking in Fig. 1. This results from two facts. First, there is a near equivalence between the Saros (223 sm = 242 dm) and a shorter 47-month eclipse cycle (47 sm = 51 dm) that is also attested in the Babylonian material. Second, the 47-month cycle is contained an even number of times (five times) in a 235-month Metonic cycle. Eclipse possibilities occurring near the descending node are labeled, sequentially in time, D_1, D_2, D_3, D_4, with D_1 the most northerly. (We will explain below how we know which is most

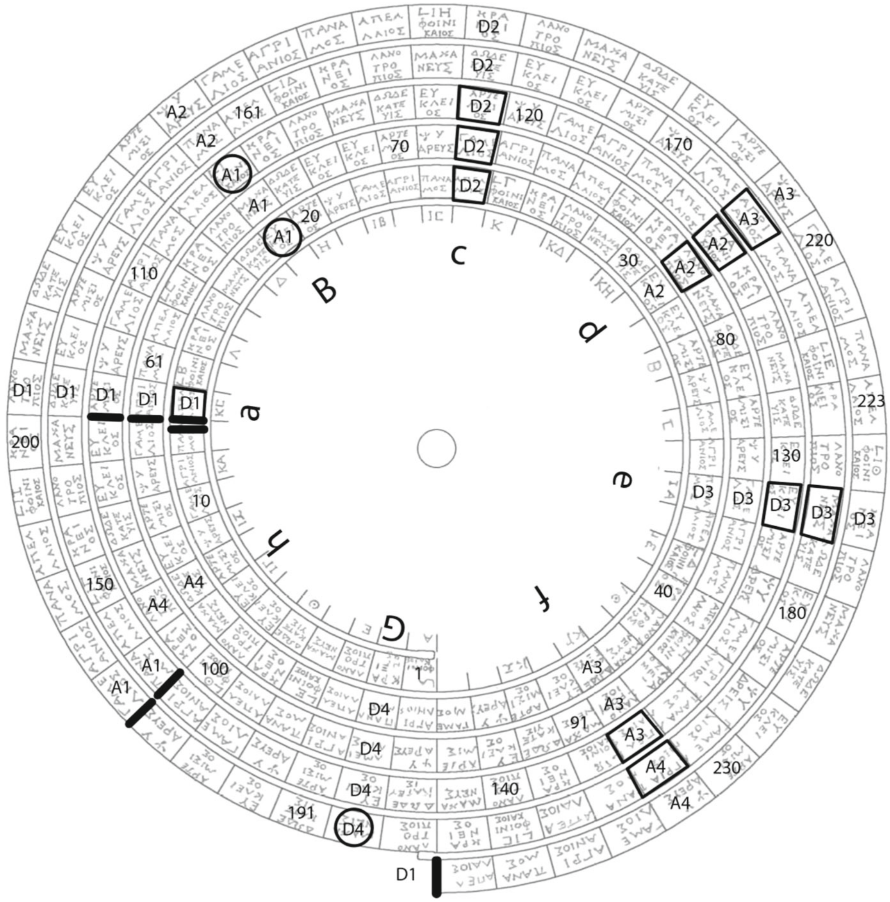
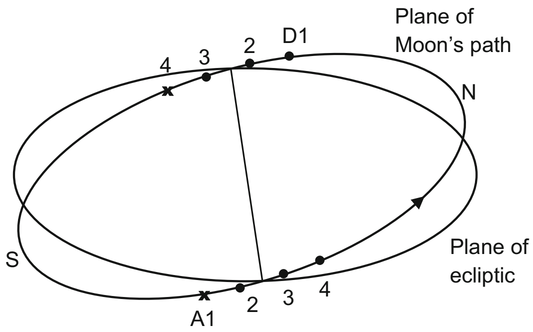


Fig. 1 Solar eclipses from the Saros dial, displayed on the Metonic calendar spiral. The actually preserved solar eclipse notices are indicated by parallelograms. The particular reconstruction of the Babylonian 8-8-7-8-7- pattern used in this figure is Solar Pattern 7. EPs called for by Pattern 7 but known to be missing from the Saros dial are indicated by circles (underlying drawing of the Metonic calendar dial© Tony Freeth 2008, reproduced by permission)

northerly). Eclipse possibilities occurring near the ascending node are labeled A_1, A_2, A_3, A_4, with A_1 being the most southerly. Note that the orientations of the radial spokes are preserved through the first two 8- groups (which are 47 months each); but shifts in absolute position are introduced by the 7- groups. We will deal with the shifts in detail below. But consideration of the first two turns already suggests a start on a simple omission rule: Eclipses in radial columns D_4 and column A_1 are apparently omitted.

To see which eclipse possibilities are northerly and which are southerly, we compare Fig. 1 with Fig. 2, which is a perspective view of the ecliptic and the Moon's inclined orbit, with N and S marking the northern and southern limits of the Moon's orbit. We have chosen to begin reckoning with the D eclipse (D_1) beginning the 8-8-. This turns out (as we will show below) to be the most northerly of four successive eclipses at the D node. The eclipse possibilities in Fig. 2 occur in the following time order (with the

Fig. 2 The pattern of included • and omitted **x** solar eclipses at each node for a 47-month cycle



omitted eclipses underlined): $D_1\underline{A}_1D_2A_2D_3A_3\underline{D}_4A_4D_1\underline{A}_1D_2A_2D_3A_3\underline{D}_4A_4$. Eclipses A_1 and D_4 are omitted because they are the most southerly at each node. (Others might possibly be omitted, too, if they are too southerly, but these two types are clear).

Now, in Fig. 1, we label the radial columns aBcdefGh, where the omitted columns are in capital letters. Between an omitted EP at B and an omitted EP at G, four EPs will intervene. But, between omitted EPs at G and B, there will be only two. In Fig. 2, we see that there are two eclipses between the omitted eclipses D_4 and A_1, but four eclipses between A_1 and D_4. By comparison, then, column B of Fig. 1 corresponds to A_1 in Fig. 2, and column G to D_4. Thus, a large amount of ambiguity has been removed from the eclipse pattern on the Saros dial of the Antikythera mechanism. We know which eclipses go with each node and, for each node, which are the most northerly and most southerly eclipses.

Conclusion of Sect. 7: It was shown by Freeth et al. that some of the solar eclipses that would be predicted by the 8-8-7-8-7- scheme on the AM were omitted. These omitted eclipses can be used to establish a classificatory scheme. We denote by D_1 the northernmost solar eclipse at the descending node in each diagonal sequence. D_2, D_3 (and sometimes D_4) label increasingly southerly eclipses in the same diagonal sequence. A_1 is the most southerly eclipse at the ascending node in a diagonal sequence. A_2, A_3 (and sometimes A_4) are progressively more northerly. Eclipses of types D_4 and A_1 were omitted. The pattern of omitted eclipses allows us to establish which eclipses go with each node and, for each node, which are the most northerly and the most southerly eclipses.

8 Justification of the omission rule for the 47-month cycle

We shall make the argument first using the 47-month cycle. (Later, we shall see what needs to be modified to use the Saros instead). The most important parameter is the ratio $r = 51/47$, since 51 draconic months = 47 synodic months. Let us begin at the first eclipse D_1 in column 1 of Table 4, and suppose, for the sake of example, that the mean conjunction occurs 13.40° clockwise (north) of the node. Then, at D_2, when 12 synodic months have elapsed, the number of draconic months elapsed will be $12\,r = 12 \times 51/47 = 13\,1/47$ dm, or 1/47 dm over and above complete draconic

Table 4 Mean solar eclipse possibilities at the descending node in a 47-month cycle

EP	Month number N_i	Time since D_1 $\Delta t = (N_i - N_1)\ (51/47)$	Angular distance from descending node $\phi_d = -360\ \text{frac}(\Delta t) + 13.4$
D_1	13 syn. mo.	0 drac. mo.	13.40°N
D_2	25	13 1/47	5.74°N
D_3	37	26 2/47	1.92°S
D_4	49	39 3/47	9.58°S
D_5	60	51	13.40°N

revolutions with respect to the node. Thus, mean conjunction D_2 will occur $360°/47 = 7.66°$ beyond (counterclockwise of) D_1. That is, D_2 will occur at a nodal distance of $13.40° - 7.66° = 5.74°$ north of the node. The situation is represented in Fig. 2.

The D_3 conjunction will occur 26 2/47 draconic revolutions after D_1 and thus $2 \times 360°/47$ counterclockwise from D_1. The D_4 conjunction will occur $3 \times 360°/47$ counterclockwise from D_1, as shown in Table 4 and illustrated in Fig. 2. Thus, as we move to successive conjunctions at the same node, they occur progressively farther counterclockwise by steps of about 7.66°, corresponding to time differences of 1/47 dm. But at D_5, because there is only an 11-month interval (instead of a 12-month interval), the time difference is only $11 \times 51/47$ dm $= (12 - 3/47)$ dm, corresponding to an angular shift of −22.98°, which exactly cancels the accumulated displacements of the three preceding steps. Thus, D_5 occurs at the same position with respect to the node as D_1, and the cycle of 47 complete synodic months starts over.

Consider now the eclipse possibilities at the ascending node. The first eclipse in radial column A_1 of Fig. 1 occurs six synodic months, and therefore, 6 24/47 = 6.5106 draconic months, after D_1. We may discard the six whole draconic revolutions; and the 0.5 merely tells us that we are 180° around the circle, near the other node. A_1 therefore occurs 0.0106 draconic months, or 3.83°, counterclockwise from the point diametrically opposite D_1, which puts A_1 some 9.57° (=13.40 − 3.83) south of the ascending node, as shown in Fig. 2 and as tabulated in Table 5. Successive eclipses A_2, A_3, A_4 occur at time intervals of 13 1/47 draconic months. But eclipse A_5 occurs at an interval of only 11 synodic months (rather than 12) and thus at an interval of $11 \times 51/47$ dm $= (12 - 3/47)$ dm, which restores the balance. That is, A_5 occurs precisely at the same nodal elongation as A_1.

In the case of the 47-month cycle, because the nodal elongations repeat after each group of eight EPs, the simple rule of omitting the eclipses of types D_4 and A_1 would be entirely equivalent to a more complex prescription based on nodal distances. We could suppose that eclipse possibilities falling less than, say, 17° north of the node are assumed actually to occur, while those falling more than 8° south of the node are excluded.

Conclusion of Sect. 8: In a 47-month cycle, the omission of solar eclipses D_4 and A_1 would be rigorously equivalent to a cutoff of southernmost eclipses based on nodal elongation.

Table 5 Mean solar eclipse possibilities at the ascending node in a 47-month cycle

EP	Month number N_i	Time since D_1 $\Delta t = (N_i - N_1)$ (51/47)	Angular distance from ascending node $\phi_a = -360\ \text{frac}(\Delta t) + 13.4 + 180$
A_1	19 syn mo.	6 24/47 drac. mo.	9.57°S
A_2	31	19 25/47	1.91°S
A_3	43	32 26/47	5.75°N
A_4	55	45 27/47	13.41°N
A_5	66	51	9.57°S

9 Justification of the omission rule for the 8-8-7-8-7- Saros scheme

We explained the omission rule for solar eclipses on the basis of the 47-month eclipse cycle, as this is very simple. Now, we apply the same ideas to the more accurate Saros cycle. Here, the crucial parameter is $s = 242/223 (\approx 1.085202)$, as 242 dm = 223 sm.

Five successive 47-month cycles would have given us a total of 40 eclipses distributed over 235 months. The Saros cycle is an improvement that distributes 38 eclipses over 223 months. So now we must investigate the distribution of the eclipses between the two nodes in the 8-8-7-8-7- Saros scheme. It is convenient to begin reckoning from the first solar eclipse in the double-8. We call the month of this eclipse month 1, and we assign it to node D.

The eclipse possibilities must therefore be distributed as in Table 6. Eclipses alternate, of course, between the D and A nodes. The groups of 8 and 7 eclipses are marked off by the heavy lines. In each case, the vertical stroke shows where a 5-month interval goes. In the list of D eclipses, we have four successive sequences of 4 eclipses, followed by a final sequence of 3 (in the second 7- group). (These sequences of 4 or 3 eclipses at the same node are what we have called "diagonal sequences"). Thus, the total number of eclipses is 19. In the list of A eclipses, the sequence of 3 falls in the middle of the 8-8-7-8-7- pattern (in the first 7), so again we have a total of 19. The pattern is symmetrical, with each node giving up one eclipse in one of the 7- groups.

Now, when one D eclipse follows another at an interval of 12 synodic months, the elapsed time measured in draconic months is

$$\Delta T_{12} = 12 \times 242/223 = 12(1 + 19/223) = 12 + 228/223 = 13\,5/223\,\text{dm}.$$

Thus, in Table 7, if the eclipse of month 1 occurs at a certain time −T, measured in draconic months with respect to the descending node, then the next eclipse will occur at $-T + 5/223$, the next one at $-T + 10/223$, and the next after that at $-T + 15/223$.

However, at an 11-synodic-month interval (e.g., between months 37 and 48), the elapsed time measured in draconic months is

$$\Delta T_{11} = 11 \times 242/223 = 11(1 + 19/223) = 11 + 209/223 = 12 - 14/223\ \text{dm},$$

Table 6 Distribution of eclipses between the two nodes in an 8-8-7-8-7- Saros scheme, with the first eclipse of the double 8-group assigned to the descending node D

D	A			
1	7		D1	A1
13	19	8	D2	A2
25	31		D3	A3
37	43		D4	A4
48	54		D1	A1
60	66	8	D2	A2
72	78		D3	A3
84	90		D4	A4
95	101		D1	A1
107	113	7	D2	A2
119	125		D3	A3
131	136		D4	A1
142	148	8	D1	A2
154	160		D2	A3
166	172		D3	A4
178	183		D4	A1
189	195	7	D1	A2
201	207		D2	A3
213	219		D3	A4
224	230		D1	A1

Table 7 Mean solar eclipse possibilities at the descending node in the Saros scheme

Synodic month	Time w/r to D node (draconic months)	Angular distance from D node (°)
1	$-T$	$360\,T$
13	$-T+5/223$	$360\,(T-5/223)$
25	$-T+10/223$	$360\,(T-10/223)$
37	$-T+15/223$	$360\,(T-15/223)$
48	$-T+1/223$	$360\,(T-1/223)$
60	$-T+6/223$	$360\,(T-6/223)$
72	$-T+11/223$	$360\,(T-11/223)$
84	$-T+16/223$	$360\,(T-16/223)$
95	$-T+2/223$	$360\,(T-2/223)$
107	$-T+7/223$	$360\,(T-7/223)$
119	$-T+12/223$	$360\,(T-12/223)$
131	$-T+17/223$	$360\,(T-17/223)$
142	$-T+3/223$	$360\,(T-3/223)$
154	$-T+8/223$	$360\,(T-8/223)$
166	$-T+13/223$	$360\,(T-13/223)$
178	$-T+18/223$	$360\,(T-18/223)$
189	$-T+4/223$	$360\,(T-4/223)$
201	$-T+9/223$	$360\,(T-9/223)$
213	$-T+14/223$	$360\,(T-14/223)$
224	$-T$	$360\,T$

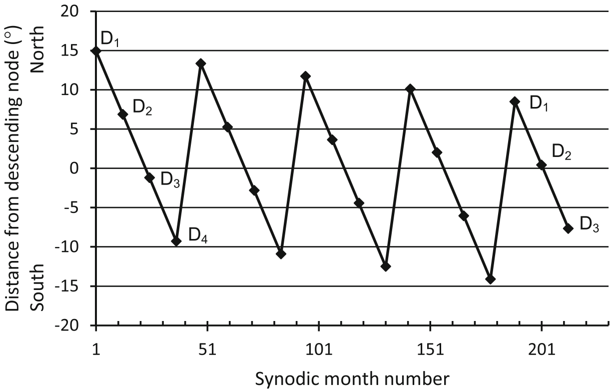


Fig. 3 Mean eclipses near the descending node in a Saros scheme: pattern of elongations from the descending node. Each sequence of 3 or 4 eclipses at the same node and lying on the same line segment is called a "diagonal sequence"

so the accumulated difference of 15/223 is almost canceled. The fraction of a draconic month by which the mean conjunction falls before or after the D node is given in the second column. The angular distance, in degrees, of the mean eclipse from the D node is then given in column 3. Positive angles are north of the node, and negative angles are south of the node.

In Fig. 3, we display, as an example, numerical values for the mean elongation from the descending node for the case $T = 18.5/446$. (This value of T makes the maximum northward and southward elongations from the node approximately equal, for the case of the descending node as well as the case of the ascending node). It is easy to see in Table 7 (or in Fig. 3) that the Saros scheme guarantees that, of all the solar eclipses occurring at the D node, the most northerly will be the D_1 that begins the double 8-. And in this figure, it is finally clear why we call the sequences of 4 or 3 eclipses at the same node "diagonal sequences." As can be seen, the prescription to omit all the D_4 eclipses will indeed remove the southernmost ones and only those. It is conceivable that the D_3 in the final sequence of three eclipses was also omitted; but we do not need this result for purposes of dating. It is enough to establish that omission of the D_4 eclipses is consistent with the evidence of Fig. 1. But the true omission rule might actually be to omit the southernmost eclipse of each diagonal sequence.

Consider now the solar eclipses at the ascending node. The eclipse in synodic month 7 occurs 6 synodic months after the eclipse of month 1. Expressed in terms of the draconic month the elapsed time amounts to

$$\begin{aligned}\Delta T_6 &= 6 \times 242/223 = 6(1 + 19/223) = 6 + 114/223 = 6 + 228/446 \\ &= 6^1/_2 + 5/446\,\mathrm{dm}.\end{aligned}$$

We may ignore the 6 whole draconic months. And $^1/_2$ draconic month just takes us from the descending to the ascending node. Thus, if the eclipse of month 1 occurs at time $-T$ with respect to the D node, eclipse 7 will occur at time $-T + 5/446$ with respect to the A node. Then, successive eclipses at the A node occur at time intervals of

Table 8 Mean solar eclipse possibilities at the ascending node in the Saros scheme

Synodic month	Time w/r to A node (draconic months)	Angular distance from A node (°)
7	−T + 5/446	360 (−T + 5/446)
19	−T + 5/446 + 5/223	360 (−T + 5/446 + 5/223)
31	−T + 5/446 + 10/223	360 (−T + 5/446 + 10/223)
43	−T + 5/446 + 15/223	360 (−T + 5/446 + 15/223)
54	−T + 5/446 + 1/223	360 (−T + 5/446 + 1/223)
66	−T + 5/446 + 6/223	360 (−T + 5/446 + 6/223)
78	−T + 5/446 + 11/223	360 (−T + 5/446 + 11/223)
90	−T + 5/446 + 16/223	360 (−T + 5/446 + 16/223)
101	−T + 5/446 + 2/223	360 (−T + 5/446 + 2/223)
113	−T + 5/446 + 7/223	360 (−T + 5/446 + 7/223)
125	−T + 5/446 + 12/223	360 (−T + 5/446 + 12/223)
136	−T + 5/446 − 2/223	360 (−T + 5/446 − 2/223)
148	−T + 5/446 + 3/223	360 (−T + 5/446 + 3/223)
160	−T + 5/446 + 8/223	360 (−T + 5/446 + 8/223)
172	−T + 5/446 + 13/223	360 (−T + 5/446 + 13/223)
183	−T + 5/446 − 1/223	360 (−T + 5/446 − 1/223)
195	−T + 5/446 + 4/223	360 (−T + 5/446 + 4/223)
207	−T + 5/446 + 9/223	360 (−T + 5/446 + 9/223)
219	−T + 5/446 + 14/223	360 (−T + 5/446 + 14/223)
230	−T + 5/446	360 (−T + 5/446)

5/223 dm, as before, with occasional resets by −14/223 dm at the 11-synodic-month intervals. See Table 8.

In Fig. 4, we graph the mean elongation from the ascending node for $T = 18.5/446$. The prescription to omit all the A_1 eclipses will indeed remove the southernmost ones and only those. It is conceivable that the A_2 following the sequence of three eclipses was also omitted; but we do not need this result for purposes of dating. It is enough to establish that omission of the A_1 eclipses is consistent with the evidence of Fig. 1).

Conclusion of Sect. 9: In a 8-8-7-8-7- Saros scheme, the omission of eclipses D_4 *and* A_1 *would cull the nine southernmost solar eclipses in each Saros cycle and would be equivalent to a cutoff based on nodal elongation. Alternatively, one could omit the southernmost eclipse of each diagonal sequence; this would eliminate the ten southernmost and would also be equivalent to a cutoff based on nodal distance.*

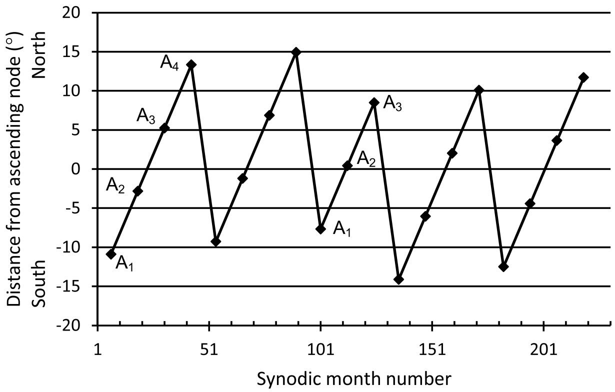


Fig. 4 Mean eclipses near the ascending node in a Saros scheme: pattern of elongations from the ascending node

10 Elimination of solar patterns 1–6

The rule that solar eclipse possibilities in position D_4 were omitted on the Antikythera mechanism turns out to be a powerful tool for restricting possibilities. In Table 9, we reprint the seven possible solar patterns from Table 2, but have cyclically shifted the entries in each column so that each column begins with the D_1 eclipse at the beginning of the double 8-. A horizontal heavy line is used to mark the beginning and end of each group of 7 or of 8 eclipses. As before, bold numbers (e.g., **13** and **25**) indicate a synodic month cell in which there is an extant solar eclipse glyph. The smaller numbers in parentheses, such as (37), indicate months for which a solar eclipse would be predicted by the 8-8-7-8-7- scheme on display in a particular column of the table. If a parenthetical number has a horizontal line through it, such as ~~(7)~~ or ~~(189)~~, this indicates that this eclipse possibility must be omitted because of empirical evidence on the Antikythera mechanism. In some cases (113, 189, and 190), this is because the month cell is preserved and it is clear that no solar eclipse possibility was engraved in it; but in other cases (7 and 19), it is because placing a solar EP here would be in conflict with the sequence of index letters.[9] Thus, the bold numbers and the lined out parenthetical numbers are the conditions that have to be satisfied by any scheme of omission of solar eclipses.

Let us consider first the eclipses at the descending node. A descending line \ through a bold-faced number indicates a D_4 eclipse that actually appears on the mechanism, contrary to the rule that such an eclipse should be omitted. Thus, in columns 3, 5, and 6, EP 137 and 184 are extant D_4. These three cases must therefore be eliminated. In column 4, EP 131 and 178 are extant D_4, so this case is eliminated. In column 1, EP 178 is an extant D_4, so this case also is excluded. Now, in column 2, the descending line\through ~~(189)~~ indicates that a D_1 is missing, contrary to the omission rule that a

[9] Cells 19 and 7 are, at least in theory, extant in fragments F and A, respectively. Cell 7 is almost impossible to see. It is also impossible to see Cell 8, or at least to read any letter. But cell 19 is clearer, and it seems reasonably clear that there is nothing there. While we cannot be sure from direct examination of these cells, the sequence of index letters is enough to decide the issue.

Table 9 Elimination of solar patterns 1–6 by means of the omitted solar eclipses

	Solar Case i			Solar Case ii		Solar Case iii	
	1	2	3	4	5	6	7
D_1	(142)	~~(189)~~	(54)	(95)	~~(7)~~	(101)	**13**
A_1	(148)	(195)	(60)	(101)	~~**13**~~	(107)	~~(19)~~
D_2	(154)	(201)	(66)	(107)	~~(19)~~	~~(113)~~	**25**
A_2	(160)	(207)	**72**	~~(113)~~	**25**	**119**	(31)
D_3	(166)	(213)	**78**	**119**	(31)	**125**	(37)
A_3	**172**	(219)	(84)	**125**	(37)	**131**	(43)
D_4	~~**178**~~	(2)	(90)	~~**131**~~	(43)	~~**137**~~	(49)
A_4	**184**	(8)	(96)	**137**	(49)	(143)	(55)
D_1	~~(189)~~	**13**	(101)	(142)	(54)	(148)	(60)
A_1	(195)	~~(19)~~	(107)	(148)	(60)	(154)	(66)
D_2	(201)	**25**	~~(113)~~	(154)	(66)	(160)	**72**
A_2	(207)	(31)	**119**	(160)	**72**	(166)	**78**
D_3	(213)	(37)	**125**	(166)	**78**	**172**	(84)
A_3	(219)	(43)	**131**	**172**	(84)	**178**	(90)
D_4	(2)	(49)	~~**137**~~	~~**178**~~	(90)	~~**184**~~	(96)
A_4	(8)	(55)	(143)	**184**	(96)	~~(190)~~	(102)
D_1	**13**	(60)	(148)	~~(189)~~	(101)	(195)	(107)
A_1	~~(19)~~	(66)	(154)	(195)	(107)	(201)	~~(113)~~
D_2	**25**	**72**	(160)	(201)	~~(113)~~	(207)	**119**
A_2	(31)	**78**	(166)	(207)	**119**	(213)	**125**
D_3	(37)	(84)	**172**	(213)	**125**	(219)	**131**
A_3	(43)	(90)	**178**	(219)	**131**	(2)	**137**
D_4	(49)	(96)	~~**184**~~	(2)	~~**137**~~	(8)	(143)
A_1	(54)	(101)	~~(189)~~	~~(7)~~	(142)	~~**13**~~	(148)
D_1	(60)	(107)	(195)	**13**	(148)	~~(19)~~	(154)
A_2	(66)	~~(113)~~	(201)	~~(19)~~	(154)	**25**	(160)
D_2	**72**	**119**	(207)	**25**	(160)	(31)	(166)
A_3	**78**	**125**	(213)	(31)	(166)	(37)	**172**
D_3	(84)	**131**	(219)	(37)	**172**	(43)	**178**
A_4	(90)	**137**	(2)	(43)	**178**	(49)	**184**
D_4	(96)	(143)	(8)	(49)	~~**184**~~	(55)	~~(190)~~
A_1	(101)	(148)	~~**13**~~	(54)	~~(189)~~	(60)	(195)
D_1	(107)	(154)	~~(19)~~	(60)	(195)	(66)	(201)
A_2	~~(113)~~	(160)	**25**	(66)	(201)	**72**	(207)
D_2	**119**	(166)	(31)	**72**	(207)	**78**	(213)
A_3	**125**	**172**	(37)	**78**	(213)	(84)	(219)
D_3	**131**	**178**	(43)	(84)	(219)	(90)	(2)
A_4	**137**	**184**	(49)	(90)	(2)	(96)	(8)

D_1 should certainly not be omitted, so this case, too, is excluded. When all the cells crossed out by descending lines \(indicating a violation of the omission rules at the descending node) are considered, it will be seen that Case 7 is the only survivor. But in Sect. 6, we showed that solar pattern 7 is only compatible with lunar pattern β. Thus, we are left with 7β as the only solution.

Some supporting evidence is also available from study of the eclipses at the ascending node. Violations of the omission rules at the ascending node are indicated by an ascending line /. It turns out that not enough of these eclipse inscriptions are preserved to provide as much discriminating power as do the eclipses at the descending node. However, cases 3, 5, and 6 are again ruled out—they each have an extant EP at an A_1 (which should be omitted) in month 13. Case 6 is also excluded because the cell for month 190 clearly has no solar eclipse glyph, but is an A_4 that, according to the omission rules, should not be omitted.

All cells that involve violation of the omission rules (at either of the two nodes) are shaded gray. It is possible that arguments could be made for a few other cells. Here, we have restricted ourselves to D_1, D_4, A_1, and A_4 eclipses, for which the rules are the clearest.

Conclusion of Sect. 10: *When the omission rules are applied, only solution 7β (from Sect.* 5*) survives as a possibility.*

11 Empirical test of the omission rules

Given that an astronomer as cogent as Ptolemy adopted an omission rule for solar eclipses occurring too far south of the node, we might consider the possibility there is something reasonable in this strategy. Therefore, we used the Javascript Solar Eclipse Explorer, on the NASA Eclipse Web Site, to determine all the solar eclipses that were visible from Babylon between the years −400 and 0 (Espenak, Solar Eclipse Explorer). There were 149 of them.

For each of these eclipses, we calculated the Moon's distance from the node at the moment of greatest eclipse. The most southern visible eclipse at the D node was 6.02° south of the node. The most southern visible eclipse at the A node was 0.2° south of the node. Now, in Fig. 4, the northernmost A_1 eclipse is around 7.7° south of the node; and in Fig. 3, the northernmost D_4eclipse is around 9.3° south of the node. Thus, it is clear that the southernmost eclipses actually visible at Babylon could not possibly be a D_4 or A_1. (The numerical boundaries of D_4 and A_1 in Figs. 3 and 4 can be modified a bit, by taking slightly different values for the parameter T in Tables 7 and 8; but the changes cannot be large, or the graphs will become grossly asymmetrical). *There simply were no D_4 or A_1 solar eclipses visible from Babylon for 400 years.* The omission rule would have performed very well.

An alternative way of seeing this is by directly categorizing eclipses in the Espenak catalog. The criterion for assigning the eclipses is as follows. Consider the D eclipses separately; usually, they are 12 months apart, but sometimes only 11. If, at the descending node, an eclipse follows an 11-month interval (at that node), it is a D_1,

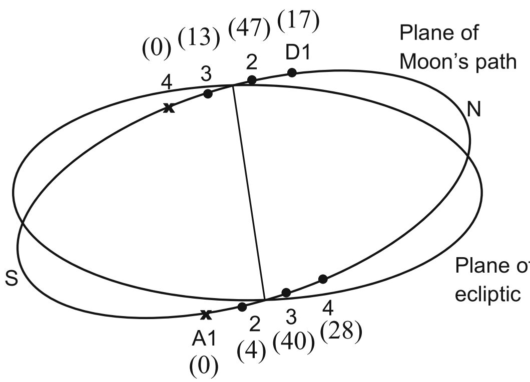


Fig. 5 Numbers of solar eclipses of each type visible from Babylon between years −400 and 0

the next is a D_2, and so on. Similarly consider the A eclipses in isolation. If, at the ascending node, an eclipse comes after an 11-month interval (at that node), it is an A_1, the next is an A_2, and so on. Then, it is an easy matter to count up the eclipses of each type. The results are displayed in Fig. 5, where the number that occurred is shown in parentheses for each type of eclipse. Again we see that there were no D_4 or A_1 eclipses were visible at Babylon. Although omission schemes of this sort have not yet been found in the Babylonian material, it is tempting to imagine that Ptolemy's omission scheme based on nodal distances is a rationalization of an earlier scheme based simply on the omission of the southernmost eclipse of each diagonal sequence, or something closely related to that.

Conclusion of Sect. 11: Omission of the D_4 and A_1 eclipses would have been an excellent empirical rule. Not a single D_4 or A_1 was visible from Babylon between years −400 and 0.

12 Candidates for epoch

Table 10 displays the solar eclipses that are the candidates for the solar eclipse in Saros cell 13 of the Antikythera mechanism. Cell 13 carries, of course, the D_1 eclipse that begins the solar 8-8- on the extant Saros dial. Dates shown in bold type come from Steele's reconstruction of the Babylonian eclipse records: These are the dates of the first solar EP in a double 8- group. (These are, of course, all eclipses of type D_1). The heavy vertical bars mark the approximate location of Babylonian recalibrations of the Saros scheme. The extreme left-hand column gives the Saros series number of each of these eclipses. The shaded gray cells show solar eclipses of type D_1 from the Espenak catalog (computed from modern theory) that could function well as the first eclipse of an 8-8-7-8-7- scheme.

Let us study how the Babylonian recalibrations occur. Take, for example, the successive Saros cycles beginning with −203 May 17, −185 May 28, and −167 June 7. Again, these are all dates of first solar eclipses in a double 8-. Since the Babylonian dates fall in the shaded cells, we can see that the Babylonian record is in good agree-

Table 10 Candidates for the date of solar eclipse D_1 in Saros cell 13 of the Antikythera mechanism

Saros series	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
38	*SEP 15* *-377*	*SEP 26* *-359*	*OCT 7* *-341*	*OCT 18* *-323*	*OCT 29* *-305*	*NOV 9* *-287*	*NOV 20* *-269*											
40	**JUL 4** **-373**	**JUL 14** **-355**	**JUL 26** **-337**	**AUG 5** **-319**	**AUG 16** **-301**	**AUG 27** **-283**	*SEP 6* *-265*	*SEP 17* *-247*	*SEP 27* *-229*	*OCT 7* *-211*								
42	*APR 23* *-369*	*MAY 3* *-351*	*MAY 14* *-333*	*MAY 25* *-315*	*JUN 5* *-297*	*JUN 16* *-279*	**JUN 26** **-261**	**JUL 7** **-243**	**JUL 18** **-225**	*JUL 29* *-207*	*AUG 9* *-189*	*AUG 20* *-171*	*AUG 31* *-153*	*SEPT 11* *-135*	*SEP 22* *-117*			
44			*MAR 2* *-329*	*MAR 13* *-311*	*MAR 24* *-293*	*APR 3* *-275*	*APR 15* *-257*	*APR 25* *-239*	*MAY 6* *-221*	**MAY 17** **-203**	**MAY 28** **-185**	**JUN 7** **-167**	**JUN 19** **-149**	**JUN 29** **-131**	*JUL 9* *-113*	*JUL 19* *-95*	*JUL 29* *-77*	
46									*FEB 22* *-217*	*MAR 4* *-199*	*MAR 15* *-181*	*MAR 26* *-163*	*APR 6* *-145*	*APR 16* *-127*	**APR 28** **-109**	**MAY 8** **-91**	*MAY 18* *-73*	*MAY 28* *-55*
48											*JAN 2* *-177*	*JAN 12* *-159*	*JAN 24* *-141*	*FEB 3* *-123*	*FEB 14* *-105*	*FEB 25* *-87*	**MAR 8** **-69**	**MAR 18** **-51**
50																*DEC 13* *-84*	*DEC 25* *-66*	*JAN 4* *-47*

A number in the left column gives the Saros series number of the eclipses in the particular horizontal row. A date in **bold** indicates the starting eclipse of a double-8 group from the Babylonian records. Heavy bars show approximate locations of Babylonian recalibrations. Shaded cells indicate eclipses that, to judge by the Espenak data base, could function well as a D_1 eclipse to start a double-8 group. The letters in the top row will be used to identify particular columns of the table

Fig. 6 Insertion of a 47-month cycle of 8 eclipses during the transition from an old to a new Saros calibration

old calibration	*8*	*7*	<u>**8**</u>	**8**	**7**	**8**	**7**	*8*
new calibration				**8**	**8**	**7**	**8**	**7**

ment with the phenomena for this stretch. But by year −149, solar Saros series 44 had ended—this was no longer an actually occurring eclipse. The Babylonians, however, continued to run their 8-8-7-8-7- scheme without modification for two more Saros cycles. Then, there was a change of calibration—with the eclipse of −109 April 28 now serving as the first eclipse of a double 8- (as opposed to the "eclipse" of −113 July 19, which has been abandoned).

The recalibration occurs in the following way (see Fig. 6). In the old calibration, the last two eclipse groups of an 8-8-7-8-7- cycle are marked in italic: *8, 7*. The next 8-8-7-8-7-, shown in bold, is scheduled to begin. However, instead, a single 8- is inserted on its own as a single 47-month bridge (shown underlined, <u>**8**</u>). Because the 8-eclipse, 47-month cycle is a good approximation to the full 223-month Saros, this does not disrupt the alternation of eclipses between the two nodes, nor affect the $D_1A_1D_2A_2\ldots$ sequence. After the bridging group (<u>**8**</u>) has been completed, the new calibration begins with the first eclipse of an 8-8-7-8-7-. The effect of the recalibration is to delay the start of the next 8-8-7-8-7- by 47 months. At least after −250, a recalibration may often be recognized in Steele's table by a run of three 8- groups in a row. With the particular Babylonian recalibration under discussion, −109 April 28 is the first eclipse of a double 8-, and the new calibration is again in good agreement with the eclipse phenomena. In a number of cases, the insertion of a delaying 47-month cycle is quite clear in Steele's tables.[10] In other cases, the situation is a bit murky in the course of the recalibration; but once it is over, it is easy to see that the date of the newly functioning 8-8- has indeed been delayed by 47 months.

It should be noted how powerful are the simplifying effects of the omission of the southernmost solar eclipses and the Babylonian connection. Because of the former, we can identify the ascending and descending nodes and can unambiguously characterize the eclipse in month 13 on the mechanism's Saros dial as a northernmost eclipse occurring at the descending node. Because of the later, we then have a restricted family of candidates for the eclipse of month 13. These are shown in bold in Table 10, though one should probably also keep under consideration eclipses that occurred in the same Saros series shortly after a recalibration (non-bold eclipses just to the right of the end of a line of bold eclipses). For example, suppose the Babylonians were running Saros series 42 for their D_1 eclipse beginning the 8-8-. A recalibration occurred, and they went over to Saros series 44. But it is conceivable that a Greek astronomer did not have up-to-date information and used the eclipse of −207 Jul 29 (instead of moving over to the eclipse of −203 May 17) as his D_1 beginning the 8-8-.

[10] For example, the solar recalibrations that took place in −387 and in −265.

Conclusion of Sect. 12: We assume, for purposes of investigation, that the D_1 solar eclipse at the start of the 8-8- on the Saros dial of the Antikythera mechanism is one of those shown in bold in Table 10. These are eclipses that served the same function in the Babylonian record. Or, if the maker of the AM were using a slightly out-of-date Babylonian Saros table, the solar eclipse month 13 could be one of the eclipses in italic, located just to the right of a sequence of bold eclipses.

13 Lunar and solar equations of center

On the Antikythera mechanism, eclipses of the same kind (solar or lunar) come at 5- or 6-month intervals. The mean synodic month T_S is about 29.53085 days[11], so a 5-month interval represents 147.654 days, or about 15.7 h over and above whole days. A 6-month interval represents 4.4 h over and above whole days. Thus, the local time of day of an eclipse should advance by either 15.7 h or by 4.4 h over its predecessor, owing to the mean motions alone. Departures of the recorded eclipse times from this simple pattern can be taken as signs that the solar and/or the lunar equation of center might be built into the predictive pattern. Solar and lunar equations could be embedded in the eclipse times in two different ways. If the eclipse times are the results of theoretical calculation, they should reflect whatever theory the ancient astronomer used to model the lunar anomaly, or solar anomaly, or both. The equation of center of the Moon could have been modeled by an epicycle or a linear zigzag function, for example. Or, if the eclipse times are based largely on observations (with some sort of interpolation for unobservable eclipses), they should reflect the actual equations of center of the Sun and Moon.

The longitude of the Sun $\lambda_\odot$ and of the Moon $\lambda_☾$ may be written

$$\begin{aligned}\lambda_\odot &= \bar{\lambda}_\odot + q_\odot\\ \lambda_☾ &= \bar{\lambda}_☾ + q_☾,\end{aligned}$$

where the overbar denotes mean quantities, which increase uniformly with time. q is the equation of center, and for either the Sun or the Moon, it may be modeled approximately by

$$q = e \sin \bar{\alpha},$$

where $\bar{\alpha}$ is the mean anomaly, defined as the excess of the mean longitude over longitude of the perigee. That is,

$$\bar{\alpha} = \bar{\lambda} - \Pi,$$

where Π is the longitude of the perigee. For the Sun, e is around 2°; and, for the Moon, around 5°.[12]

[11] We take the year to be 365 1/4 days and assume the Metonic relation 235 synodic months = 19 years.

[12] The maximum value of the Moon's equation of center is actually about 6.3°; but we are concerned with eclipses, i.e., with phenomena taking place at new and full Moon. In these situations, the equation of center is reduced by the evection by about 1.3°; thus, the effective maximum equation of center at new and full Moons is about 5°. We neglect higher order terms in the equation of center, the annual equation, and other small perturbations of the Moon's longitude.

Now, at the middle of a solar eclipse—at a true conjunction—we must have

$$\lambda_\odot = \lambda_☾$$

and thus

$$\bar{\lambda}_☾ = \bar{\lambda}_\odot + q_\odot - q_☾ \quad \text{(at mid-eclipse).}$$

Consider first the effect of $q_\odot$ acting alone (by setting $q_☾$ temporarily equal to zero). A positive $q_\odot$ tends to make $\bar{\lambda}_☾$ greater than $\bar{\lambda}_\odot$ at the time of true conjunction: The true conjunction therefore occurs after (later than) the mean conjunction. Consider now the effect of $q_☾$ alone. A positive $q_☾$ tends to make $\bar{\lambda}_☾$ less than $\bar{\lambda}_\odot$ at the time of true conjunction: The true conjunction therefore occurs before (earlier than) the mean conjunction.

The mean motion of the Moon with respect to the Sun is roughly 12.2°/day. Thus, the effect of the solar anomaly is to make the true conjunction come later than the mean conjunction by about $(2°)/(12.2°/\text{day}) = 3.9\,\text{h}$ when the solar equation is positive and at its peak value. (Here, for simplicity, we take the relative velocity of the Moon and Sun to be constant. In Sect. 19, we shall see how to take its variability into account). The effect of the lunar anomaly is to make the true conjunction come earlier than the mean by about $(5°)/(12.2°/\text{day}) = 9.8\,\text{h}$ when the lunar equation is positive and at its peak value. Alexander Jones made a study of the eclipse times in the glyphs of the Saros dial, successfully detected the lunar anomaly, and found that the Moon was very nearly at apogee at the full Moon of Saros cell 1. According to Jones, the solar anomaly did not show up (Jones, personal communication).

To make our own study, we proceed as follows. We will reckon time from the opposition (full Moon) of month 1 on the Saros dial. We assume that the mean opposition of month 1 occurs at a local hour $\bar{h}_0$ (a time of day), whose value is not yet known.[13] Suppose that in some later month, there is a lunar or solar eclipse that, according to the mechanism, occurs ΔM synodic months after the opposition of month 1. For lunar eclipses, ΔM will be an integer. For solar eclipses, ΔM will be an integer plus one half. Now, if the Moon and Sun both moved uniformly, the *expected mean time* of day (EMT) at which the second eclipse occurs would be given by

$$\text{EMT} = \bar{h}_0 + 24\,\text{frac}(T_\text{S}\Delta M),$$

where T_S is the length of the synodic month in days and frac() denotes the fractional part of the argument (i.e., with the integral part discarded). For example, since T_S is about 29.53085^d, two eclipses that take place 12 synodic months apart should be separated, on the average, by $29.53085^\text{d} \times 12 = 354.3202^\text{d}$. The fractional part of this is $0.3202^\text{d} = 8.885^\text{h}$. Thus, the second eclipse would take place 8.9 h later in the day, if there were no effects of the solar or lunar anomaly.

13 By the instant of opposition, we mean instant when the Moon's mean longitude is equal to the Sun's mean longitude + 180°. $\bar{h}_0$ is the local time of day at which this happened. The time of true opposition would be h_0, which differs from $\bar{h}_0$ due to the contributions of the lunar and solar equations.

Let GT (for *glyph time*) be the local time of the eclipse indicated by the eclipse glyph. (We will discuss below how we turn the glyph times into 24-h time). Then, the *advance or delay in time* (ADT) of the recorded glyph, in comparison with what would be expected if only mean motions were involved, is

$$\mathrm{ADT} = \mathrm{GT} - \mathrm{EMT}.$$

The ADT for each eclipse thus depends on the glyph time GT for that eclipse, and the number ΔM of mean synodic months elapsed since the full Moon of month 1. However, ADT also contains the unknown parameter $\bar{h}_0$ (the local time of day of the mean opposition of month 1).

Now, as explained above, if the ADT values of the eclipses are due to the lunar and/or the solar equation of center, they can be modeled by sinusoidal fitting functions. Consider the lunar anomaly. The argument of the sine function is the mean anomaly of the Moon—how far the mean Moon is past the perigee. Thus, the argument, expressed in radian measure, is

$$\bar{\alpha} = \quad 2\pi(\Delta t/T_A + \varphi_☾),$$

where T_A is the anomalistic month (the average time between successive perigee crossings) and Δt is the time elapsed between the initial moment and the moment in question. Thus, $\Delta t/T_\mathrm{A}$ is the time elapsed expressed as a fraction of the anomalistic month, while $2\pi\Delta t/T_\mathrm{A}$ is the increase in the mean anomaly (in radians) beyond its starting value. $2\pi\varphi_☾$ is the value of the mean anomaly at the initial moment. ($\varphi_☾$ itself is expressed as a fraction of an anomalistic month and must be between 0 and 1).

The case goes similarly for the Sun, except that the period involved is the tropical year of length Y. Thus, the *ADT* values can be fitted by a function of the form

$$F = -A_☾ \sin 2\pi(\Delta t/T_\mathrm{A} + \varphi_☾) + A_\odot \sin 2\pi(\Delta t/Y + \varphi_\odot),$$

where $A_☾$ and $A_\odot$ are positive and are expressed in hours. (Note their opposite signs, explained above). Δt is the time in days elapsed since the opposition of month 1, T_A is the length of the anomalistic month, and Y is the tropical year. $\varphi_☾$ is the initial value of the lunar mean anomaly, expressed as a fraction of an anomalistic month, and $\varphi_\odot$ is the initial solar mean anomaly, reckoned from the solar perigee, at the opposition of month 1, expressed as a fraction of a year. Note that $\Delta t = \Delta M\, T_\mathrm{S}$. Thus, we may put $\Delta t/T_\mathrm{A} = \Delta M\, T_\mathrm{S}/T_\mathrm{A}$. Similarly, $\Delta t/Y = \Delta M\, T_\mathrm{S}/Y$.

We form the quantity Q by computing $(\mathrm{ADT} - F)^2$ for each eclipse and summing over all the extant eclipses:

$$Q = \Sigma_\mathrm{i}\left[GT_\mathrm{i} - \bar{h}_0 - 24\,\mathrm{frac}(T_\mathrm{S}\,\Delta M_\mathrm{i}) + A_☾ \sin 2\pi(\Delta M_\mathrm{i} T_\mathrm{S}/T_\mathrm{A} + \varphi_☾) \right. \\ \left. - A_\odot \sin 2\pi(\Delta M_\mathrm{i} T_\mathrm{S}/Y + \varphi_\odot)\right]^2.$$

For each extant eclipse i, the input data taken from the mechanism are GT_i (local time of the eclipse) and ΔM_i (number of mean synodic months elapsed since the opposition of month 1). We minimize Q by varying the five unknown quantities $A_{☾}$, $A_{\odot}$, $\varphi_{☾}$, $\varphi_{\odot}$, and $\bar{h}_0$. The parameters $A_{☾}$, $A_{\odot}$, $\varphi_{☾}$, and $\varphi_{\odot}$ characterize the lunar and solar equations of center that best account for the pattern of eclipse times on the Saros dial. Note that Q contains an adjustable time $\bar{h}_0$, which can be interpreted as the local time of the instant of mean opposition in month 1. From a data fitting point of view, $\bar{h}_0$ is responsible for adjusting the mean value of ADT to approximately zero, so that the data may be fitted by the two sine functions.

We must say a word about the form in which the eclipse times are inscribed on the mechanism. First, there is a notice of whether the eclipse is of the Sun or Moon. Then, the hour is given. For an eclipse of the Sun, if the eclipse takes place in the daytime, the hour is simply noted as a number; but if the eclipse takes place at night, the hour is given along with a notation for "night." For an eclipse of the Moon, the convention is just the opposite: If the eclipse takes place in the night, the hour is noted, but no other notice is given; but if the eclipse takes place in the day, the hour is noted along with a notation for "day."

There is some question of whether the eclipse times are meant to be in seasonal or equinoctial hours. The fact that, for the 22 preserved eclipses, there is no case of an hour exceeding 12 seems to imply seasonal hours. On the other hand, the exeligmos dial provides for 0, 8, or 16 h to be added to the recorded eclipse times as we move through three Saros cycles, and these can only make sense as equinoctial hours. We shall treat the hours in the eclipse glyphs as if they were all evaluated on the equinox. Thus, we treat the day as beginning at 6 a.m. and ending at 6 p.m. So a notice for the first hour of the day is translated to 7^{h} in a 24 h clock system with the 0 h at midnight, and so on:

1st hour of day	7^{h}
6th hour of day	12
12th hour of day	18
1st hour of night	19
6th hour of night	24 or 0
7th hour of night	1
12th hour of night	6

We do not know if this is really the correct way to treat the times. However, for days within a month of the equinox, and for times within a couple of hours of noon or midnight, the error of interpretation will be negligible. For other times of year and of the day, the error of interpretation could potentially rise to 1¼ h or so (supposing, for instance, that the mechanism was built for the clime of 14½ h). In the search for the presence of a lunar or a solar equation (with amplitudes of about 10 and 4 h, respectively), the possibility of such small errors should not prevent us from proceeding.

Finally, we must point out that for some of the eclipses it is necessary to correct the glyph time by ± 24 h. Here is the reason. Let us begin from the lunar eclipse of month 20, which is marked on the mechanism as occurring at sixth hour of the night (=midnight, or 0^{h} in the 24-h clock system). Now, take some other eclipse, such as the lunar eclipse of month 137, which, according to the mechanism, takes place at the fifth hour of the day (=11^h in the 24-h system). Since the second eclipse takes place 117 synodic months later than the first one, from the mean motions alone the second eclipse should occur $117 \times 29.53085^d = 3455^d 2.62^h$ later than the first one. Thus, since the first eclipse occurred at 0^h, we would expect the second eclipse to occur at 2.62^h on the basis of the mean motions alone. However, the advance in time between the two eclipses as engraved on the mechanism is 11^h. Thus, the eclipse time has apparently advanced by $11 - 2.62 = 8.38$ h more than we would expect on the basis of the mean motions. These 8.38 h are presumably due to the change in the solar and/or lunar equation. However, we do not really know that the time of eclipse has advanced by 8.38 h. If the second eclipse occurred $3{,}454^d\ 11^h$ after the first one, or $3{,}455^d\ 11^h$ after the first one, or $3{,}456^d\ 11^h$, the ancient mechanic would have in any case inscribed the time as the fifth hour of the day. But in the first case, the advance in time from the first eclipse, over and above the effect of the mean motions, would be $8.38 - 24 = -15.62^h$; in the second case, the advance would be 8.38^h; and in the third case, it would be $8.38 + 24 = 32.38^h$. That is, we cannot tell a priori whether the effect of the lunar and solar equations has been to advance the second eclipse by 8.38 h with respect to the mean motions, to advance it by 32.38, or to retard it by 15.62. Now, the maximum lunar equation of 10 h and the maximum solar equation of 4 h together amount to a maximum of 14 h. If the first eclipse occurred early by, say, 14 h and the second occurred 14 h late, the effect of the lunar and solar equations could shift the expected time of the second eclipse by up to 28 h, over and above the effect of the mean motions. Thus, the possibility of an advance by 32.38^h may probably be ruled out. However, we cannot tell whether the eclipse time advanced by 8.32^h or was retarded by 15.62^h, over and above the effect of the mean motions.

Considerable headway can be made in determining which eclipses require a 24-h correction by comparing pairs or larger sets of eclipses for which the lunar mean anomaly is nearly the same. These would be eclipses separated by nearly a whole number of anomalistic months. Then, the effect of a change in the lunar equation is largely removed. Thus, the maximum possible time shift, over and above the effect of the mean motions, would be about 8 h—the maximum possible shift due to the solar anomaly acting alone. See "Appendix 4" for examples this sort of analysis.

Table 11 contains the fundamental data used for the analysis of the solar and lunar anomalies. The first column shows the events—eclipses of the Sun (S) and Moon (M) engraved in particular month cells of the Saros dial. We choose to reckon time from the opposition of month 1 (which was not an eclipse). The second column gives the times of day for the eclipses, as engraved on the AM—for example, for eclipse 13S, the first hour of the day. (No time of day is inscribed for the opposition of month 1). In the next few columns, we put these data into forms that are suitable for the analysis. Column 3 gives the number of synodic months elapsed from the opposition of month 1. Each lunar eclipse occurs a whole number of synodic months from this opposition. Each solar eclipse occurs a half-integral number of synodic months from our starting point. The fourth column gives the times of day of the eclipses, converted to modern 24-h time using the method described above. As explained above, for the purposes of investigation of the anomalies, it is necessary to correct some of the clock times by 24 h. The required corrections are given in column 5, and the method for making them is explained in "Appendix 4." The adjusted times are shown in the 6th column (which

Table 11 Months and times of day for eclipses of the Moon (M) and Sun (S) engraved on the Saros dial of the Antikythera mechanism. These are the fundamental data used to investigate the lunar and solar equations

Data on Saros dial		In form for data analysis			
Eclipse	Hour	Time from opposition 1 (syn. mo.)	Time of day (h)	Cor. (h)	Adjusted time (h)
1 Op		0.0	$\bar{h}_0$	0	$\bar{h}_0$
13S	1 d	12.5	7		7
20M	6 n	19.0	0		0
25S	6 d	24.5	12		12
26M	7 d	25.0	13	−24	−11
78S	1 d	77.5	7		7
79M	10 d	78.0	16	−24	−8
114M	12 d	113.0	18		18
119S	11 n	118.5	5		5
120M	6 d	119.0	12	−24	−12
125M	8 d	124.0	14		14
125S	3 d	124.5	9		9
131M	2 n	130.0	20	−24	−4
131S	9 n	130.5	3		3
137M	5 d	136.0	11	−24	−13
137S	12 d	136.5	18		18
172M	6 n	171.0	24		24
172S	12 d	171.5	18	−24	−6
178M	9 n	177.0	3		3
178S	9 d	177.5	15		15
184M	4 d	183.0	10		10
184S	1 d	183.5	7		7
190M	9 d	189.0	15	−24	−9

is the sum of columns 4 and 5). The data in columns 3 and 6 are the subject of the investigation. Note that $\bar{h}_0$, the time of day of the opposition of month 1, is an unknown parameter that must be obtained in the course of the investigation. Column 3 provides the ΔM_i appearing in the expression for Q above. Column 6 provides the GT_i.

The results of the parameter fitting are shown in Table 12. Using all 22 eclipses (both lunar and solar), we find the presence of both a lunar and a solar equation, with amplitudes in the expected ranges. Table 12 also demonstrates a reassuring stability of the parameters. Thus, if we use all 22 eclipses, or if we exclude either 2 or 4 outliers, or if we use only the 12 lunar eclipses, or weight the solar eclipses by 50 % (because, as we shall see, the timing errors for the solar eclipses are larger than for the lunar), the results do not change drastically. The amplitude of the lunar equation varies between 9.0 and 9.8 h, in the expected range. The amplitude of the solar anomaly varies between 3.0 and 3.4—close to what we would expect. $\bar{h}_0$ varies between −7.96 and −9.41 h.

Table 12 Lunar and solar parameters deduced from the eclipse data in Table 11

	$A_☾$ (h)	$\phi_☾$ (°)	$A_\odot$ (h)	$\phi_\odot$ (°)	$\overline{h}_0$ (h)
Using all 22 eclipses	9.73	191.1	3.30	151.0	−8.83
Excluding 13S and 125M	9.72	191.4	3.09	171.1	−9.39
Excluding 13S, 120M, 125M, 184M	9.11	184.7	3.03	180.4	−9.41
12 lunar eclipses only	9.84	206.0	3.27	171.1	−7.96
All 22 eclipses, Solar 50 % weight	9.80	193.6	3.41	149.5	−8.64
Omit 13S, 120M, 125M and 184M, Solar 50 % weight	9.04	184.8	3.26	175.5	−9.28

(These values of $\bar{h}_0$ are equivalent to 16:02 and 14:35—our range for the expected time of day of the opposition of month 1. As we shall see in Sect. 22.2, this is a useful result for dating).

The lunar mean anomaly (reckoned from perigee) at the opposition of month 1 comes out between 184.7° and 206.0°. These are all reasonably near the apogee of the Moon's orbit. That is, it seems that at the opposition of month 1, the Moon was near apogee. Here, we confirm a result of Alexander Jones. Jones conjectured that this might explain why the Saros dial beings with an empty, eclipseless month—the starting point was a date on which the Moon was at apogee. We certainly do not imagine that we can determine the Moon's anomaly to better than ±2 days' worth of motion, i.e., about 26°, so our results are compatible with Jones' conjecture. While Jones found no evidence of a solar equation, we do find a robust solar signal. The solar anomaly (reckoned from perigee) comes out between 149.5° and 180.4° (depending on the subset of eclipses used). Notably, these are near the solar apogee. Thus, it seems possible that the ancient mechanic was using an arithmetical template for the anomalies, which started with both solar and lunar anomalies equal to zero (measured, in ancient fashion, from apogee).

Graphs of the fits are shown in Figs. 7, 8, 9, and 10. Figures 7 and 8 give the fits with four outlier points excluded. Figures 9 and 10 show the best fits with all 22 eclipses used. In "Appendix 5," we discuss the identification of outliers. For here it suffices to note that the residuals for 13S and 125M are so great that matters would be improved if these two eclipses were changed day for night (a 12-h shift), a point already noted by Jones. We suspect a copying error in the engraving. Also, the hour inscribed for 120M has an underdot in Jones's text, indicating that the reading is not certain.

Conclusion of Sect. 13: The eclipse times inscribed on the Saros dial of the AM reveal the clear presence of a lunar equation of center and probably also of a solar equation of center. The lunar mean anomaly (reckoned from perigee) at the position of month 1 was near 191°. *The solar mean anomaly at the opposition of month 1 was between* 150° *and* 180°.

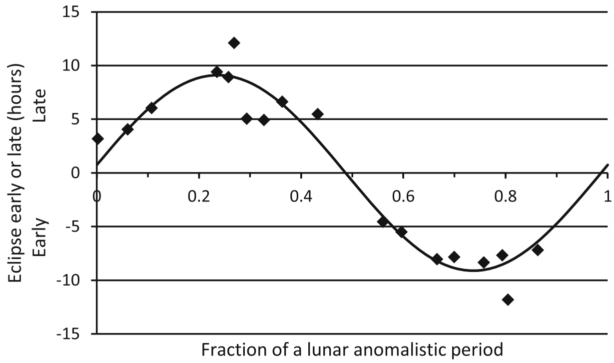


Fig. 7 The lunar equation signal in the eclipse times of the Saros dial of the Antikythera mechanism. $t = 0$ is the opposition of month 1. In this graph, 18 eclipses are used (excluding the outliers 13S, 120M, 125M, and 184M). Plotted on the vertical axis is the amount by which each eclipse comes early or late with respect to the effect of mean motions alone *minus* the solar anomaly fit. The horizontal axis gives the number of anomalistic periods elapsed, over and above complete anomalistic months, measured from the opposition of month 1. The sine curve is the best-fit lunar anomaly function with $A_{☾}$ and $\varphi_{☾}$ given by the third data row of Table 12

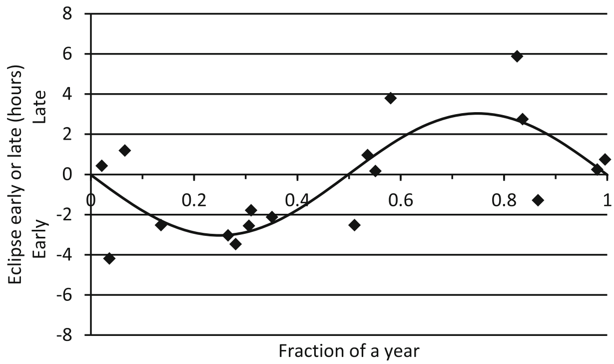


Fig. 8 The solar equation signal in the eclipse times of the Saros dial. $t = 0$ is the opposition of month 1. In this graph, 18 eclipses are used (excluding the outliers 13S, 120M, 125M, and 184M). Plotted on the vertical axis is the amount by which each eclipse comes early or late with respect to the effect of mean motions alone *minus* the lunar anomaly fit. The horizontal axis gives the number of years elapsed, over and above complete years, measured from the opposition of month 1. The sine curve is the best-fit solar anomaly function with $A_{\odot}$ and $\varphi_{\odot}$ given by the third data row of Table 12

14 Statistical test of the explanatory power of the solar fit

The fact that the lunar equation of center is present in the eclipse times inscribed on the AM seems indisputable. For the solar equation, the evidence is less clear-cut, so it is important to perform a statistical test to see how much faith should be placed in this result. Does the theoretical model with lunar equation plus solar equation provide a quantifiable explanatory advantage over a theoretical model with lunar equation only?

In this case, we have two models, a full model and a reduced model, with the reduced model nested within the full one. In the full model, there are five parameters—

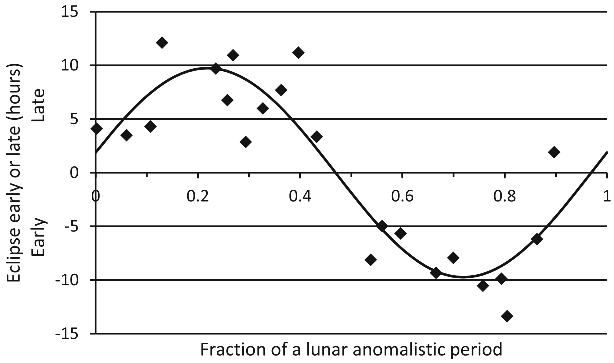


Fig. 9 The lunar equation signal in the eclipse times of the Saros dial, using all 22 eclipses. $t = 0$ is the opposition of month 1. Plotted on the vertical axis is the amount by which each eclipse comes early or late with respect to the effect of mean motions alone *minus* the solar anomaly fit. The sine curve is the best-fit lunar anomaly function with $A_\odot$ and $\varphi_\odot$ given by the top data row of Table 12

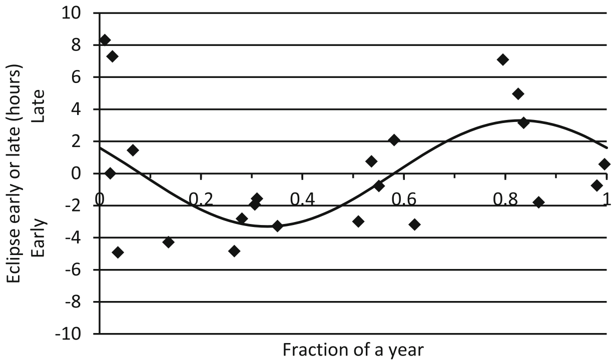


Fig. 10 The solar equation signal in the eclipse times of the Saros dial, using all 22 eclipses. $t = 0$ is the opposition of month 1. Plotted on the vertical axis is the amount by which each eclipse comes early or late with respect to the effect of mean motions alone *minus* the lunar anomaly fit. The sine curve is the best-fit solar anomaly function with $A_\odot$ and $\varphi_\odot$ given by the top data row of Table 12

a clock calibration $\bar{h}_0$ (which sets the time of day for the opposition of month 1), the amplitude and phase of the lunar equation of center, and the amplitude and phase of the solar equation. The reduced model is obtained from the full model by setting the amplitude of the solar equation of center equal to zero, in which case two parameters may be removed from the model. We shall use an F-test (named after the British statistician Ronald Fisher, who introduced a version of it in the 1920s) to assess whether the solar parameters contribute meaningfully. The null hypothesis for this type of F-test is that the full model offers no additional explanatory power over the reduced model. In other words, the null hypothesis is that the "true" underlying relationship between the recorded times of day for the eclipses and the explanatory parameters (amplitude and phase of the lunar equation of center, etc.) is that given by the reduced model; the full model with its additional parameters is unwittingly modeling as a systematic relationship some of what is actually random fluctuation.

Table 13 F-test for the explanatory power of the solar equation of center

	F	$k_F - k_R$	$n - k_F$	p
All 22 eclipses	4.27	2	17	0.031
20 eclipses, excluding 13S and 125M	5.92	2	15	0.013
18 eclipses, excluding 13S, 120M, 125M, 184M	7.52	2	13	0.0068

The F-statistic is

$$F = \frac{(\mathrm{SSR_R} - \mathrm{SSR_F})/(k_\mathrm{F} - k_R)}{\mathrm{SSR_F}/(n - k_\mathrm{F})},$$

where $\mathrm{SSR_R}$ is the sum of the squared residuals in the reduced model and $\mathrm{SSR_F}$ is the sum of the squared residuals in the full model. k_F is the number of parameters in the full model (in this case, 5) and k_R is the number of parameters in the reduced model (in this case, 3). n is the number of data points whose values we are trying to explain. The numerator of F is the reduction in the total squared error as we pass over from the reduced to the full model, divided by the number of extra parameters used. Thus, the numerator is a measure of the increase in explanatory power per extra parameter used. The denominator is the total squared error remaining in the full model per degree of freedom. If the null hypothesis were true (and the full model were therefore no real improvement on the reduced model), we would expect F to be near 1. If the new parameters of the full model do a good job of reducing the residuals, then F will be larger than 1. The larger the departure of F from unity, the more the null hypothesis is called into doubt. Tables or packaged programs can be used to determine a p-value as a function of the three values F, $k_\mathrm{F} - k_\mathrm{R}$, and $n - k_\mathrm{F}$. A p value of 0.05, for example, means that if the reduced model really were adequate, the full model would still have a probability of 0.05, simply by chance, of reducing the *SSE* by at least as much as it has.[14] Thus, a small p value provides evidence that the full model makes a significant improvement in the reduced model.

The results of the computations are shown in Table 13. As we can see, with the four outliers discarded, the p value is 0.0068. Even with the outliers included (using all 22 eclipses), the p value is only 0.03. It therefore appears likely that the inclusion of the solar equation of center provides a meaningful reduction in the residuals. However, we have no need of the solar equation to establish an epoch and shall not make deductions from the solar equation a central part of our argument. However, as we shall see in Sect. 22.1, this can be used after the epoch is established to provide some additional confirming evidence.

[14] For an introduction to the F-test, see (Ramsey and Schafer 2002, pp. 122–127, 280–285), with tables at pp. 720–727. A more mathematically detailed explanation is given in Fox (2008, pp. 200–202). Functions for calculating the p value from the F-statistic are provided in *Excel* and other commonly available programs.

Conclusion of Sect. 14: The solar equation of center embedded in the eclipse times on the Saros dial of the AM is probably meaningful. The probability that mere chance could reduce the residuals by as much as or more than does the solar fit is only around 3 %, or <1 %, depending on whether all eclipses are used or some outliers are excluded.

15 Determination of the Saros series number of the solar eclipse of month 13

We have shown that the Saros dial of the AM entails that the lunar mean anomaly was around 191° (reckoned from perigee) at the opposition of month 1. Because we do not imagine that we can determine the anomaly to a better precision than about two days' worth of motion, we shall take it to be 191° ± 26°. When used in conjunction with the Babylonian recalibrations, this provides a dating tool of considerable power. This is because of the discontinuous jump in the lunar mean anomaly produced by a Babylonian recalibration. A Saros cycle contains approximately a whole number of synodic months, draconic months, and anomalistic months. Thus, two eclipses in the same Saros series will have similar values of the lunar mean anomaly. But during a recalibration, a 47-month period is slipped in. The 47-month period contains approximately a whole number of synodic months and draconic months, but does not contain a whole number of anomalistic months. This is the source of the discontinuity of the lunar mean anomaly over a recalibration.

Our candidates for the solar eclipse of month 13 (which begins the 8-8- and therefore must be a D_1) are shown in bold in Table 10. Thus, we expect that the solar eclipse of month 13 belongs to one of the solar Saros series 40, 42, 44, 46, or 48. The bold-faced eclipses are those that occur in the Babylonian record as starting a double 8-. Of course, it is also possible that a Greek mechanic did not get the word about the latest Babylonian recalibration and continued to use eclipses predicted for a running Saros series, even after the Babylonians had abandoned it. Thus, in Table 10, we could consider a few eclipses that continue a horizontal line immediately after the end of the bold type and the heavy vertical bar marking a recalibration.

We compute from modern theory the value of the lunar mean anomaly at the middle of each solar eclipse belonging to these Saros series, for eclipses from the middle of the fourth to the middle of the first century BCE.[15] We assume, for each in turn, that one of these eclipses is the eclipse of month 13. Then, it is an easy matter to roll the calculated lunar mean anomaly back by 12½ synodic months, to determine the value that the mean anomaly would have had at the opposition of month 1, if the eclipse being examined really were the solar eclipse of month 13.

[15] For the middle of the eclipse, here, more precisely, is what we used: For eclipses that actually occurred, we used the time of greatest eclipse from the Espenak solar eclipse catalog. For eclipses predicted by the Babylonians which did not really occur, we use the time of true conjunction, from (Espenak, Six Millennium Catalog of Phases of the Moon).

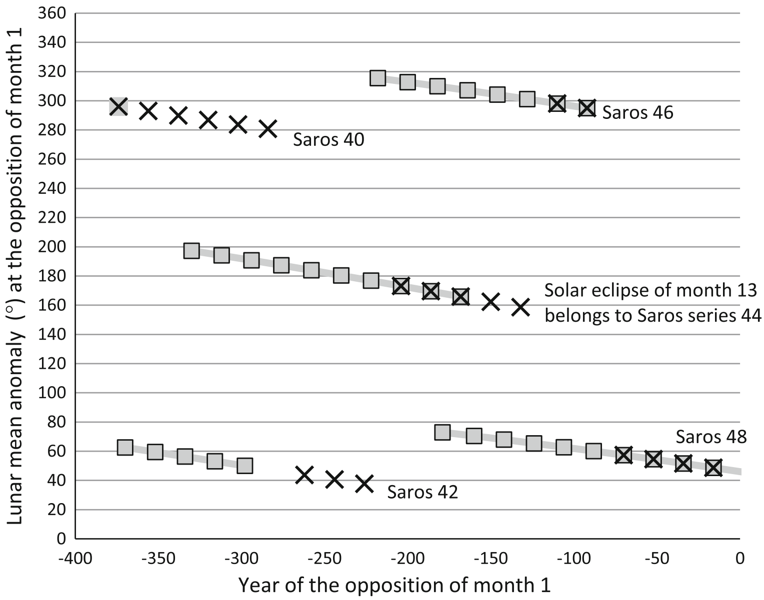


Fig. 11 Demonstration that the solar eclipse of month 13 belongs to lunar Saros series 44. Plotted on the vertical axis is the lunar mean anomaly that must obtain at the (true) opposition of month 1, for various choices of the identity of solar eclipse 13. For example, for Saros series 40, the six x's correspond to the six bold-faced eclipses for Saros series 40 in Table 10. In the Babylonian record, each of these eclipses served as the D_1 eclipse beginning a 8-8-. The gray boxes correspond to eclipses that could have worked well (according to modern data in the Espenak catalog) as the D_1 eclipse beginning a 8-8-. Now, the lunar mean anomaly is known to have been in the vicinity of 180° for the opposition of month 1. The solar eclipse of month 13 must therefore have belonged to solar Saros series 44

In Fig. 11, we see a graph showing the values that the lunar mean anomaly must have at the opposition of month 1, under the assumption that the solar eclipse of month 13 corresponds to any of the eclipses in a given Saros series. For example, we compute the value of the lunar mean anomaly at the opposition of month 1, under the assumption that the solar eclipse of month 13 belongs to solar Saros series 46 (including eclipses on −217 Feb 22, −199 Mar 4, …, −109 Apr 28, and −91 May 8). As can be seen, the possible values of the mean anomaly for the opposition of month 1 form a slowly decreasing sequence. Each **x** indicates an eclipse used as a D_1 beginning the double 8- in the Babylonian record. Each gray square indicates an eclipse that would function well as such a D_1 according to the Espenak catalog. As can be seen, assuming that the eclipse of month 13 belongs to solar Saros series 44, or 42, or 46 results in quite different sequences of possible values for the mean anomaly at the opposition of month 1. Since the value of the lunar mean anomaly is known to be near 180° for the opposition of month 1, it is clear that the solar eclipse of month 13 must belong to solar Saros series 44. The large jump in the mean anomaly from one Saros series to the next at the time of the recalibration (a discontinuity of 134°) makes only one solution possible. So, now we are left with a much smaller number of eclipses that are candidates for the solar eclipse of month 13, and they all belong to solar Saros series 44 or its continuation.

Table 14 Candidates for solar eclipse 13, all from solar Saros series 44

Group	Candidates for eclipse 13 in solar Saros series 44	Resulting date for opposition in month 1
1	−293 Mar 24	−294 Mar 20
2	−275 Apr 3	−276 Mar 30
3	−257 Apr 15	−258 Apr 10
1	−239 Apr 25	−240 Apr 21
2	−221 May 6	−222 May 2
3	**−203 May 17**	−204 May 12
1	**−185 May 28**	−186 May 23
2	**−167 June 7**	−168 June 3
3	**−149 June 19**	−150 June 14
1	**−131 June 29**	−132 June 24
2	−113 July 9	−114 July 5
3	− 95 July 19	−96 July 16

Those in bold served as the leadoff eclipses of an 8-8- in the Babylonian record

Conclusion of Sect. 15: The solar eclipse of month 13 on the Saros dial of the Antikythera mechanism belongs to solar Saros series 44.

16 Absolute time of eclipses

In Table 14, we display the dates the eclipses of Saros 44 that are candidates for the solar eclipse of month 13, together with the resulting dates for the opposition of month 1. (Those shown in bold were actually used by the Babylonians to begin a double 8- group). For all the eclipses in group 1, the time of day of the eclipse should be roughly the same. The eclipses in group 2 should come roughly 8 h later in the day and in group 3 should come about 16 h later in the day. We refer to this as the "rule of 8 h," which is a well-known predictive tool from Babylonian and Greek astronomy. When the extant eclipse times engraved on the Saros dial are compared with the actual times of these candidate eclipses, it is possible to rule out two groups of the three.

The actual times of greatest eclipse are given by Espenak, in terms of dynamical time (TD). (Espenak, Five Millennium Catalog of Solar Eclipses) and (Espenak, Five Millennium Catalog of Solar Eclipses). We convert to theoretical values of local "Antikythera Mechanism Time" (AT) for each eclipse by applying ΔT (due to the deceleration of the Earth's rotation) and a correction for change of meridian from Greenwich to the middle of the Greek cultural zone:

$$\mathrm{AT} = \mathrm{DT} - \Delta T + 1^{\mathrm{h}}35^{\mathrm{m}}.$$

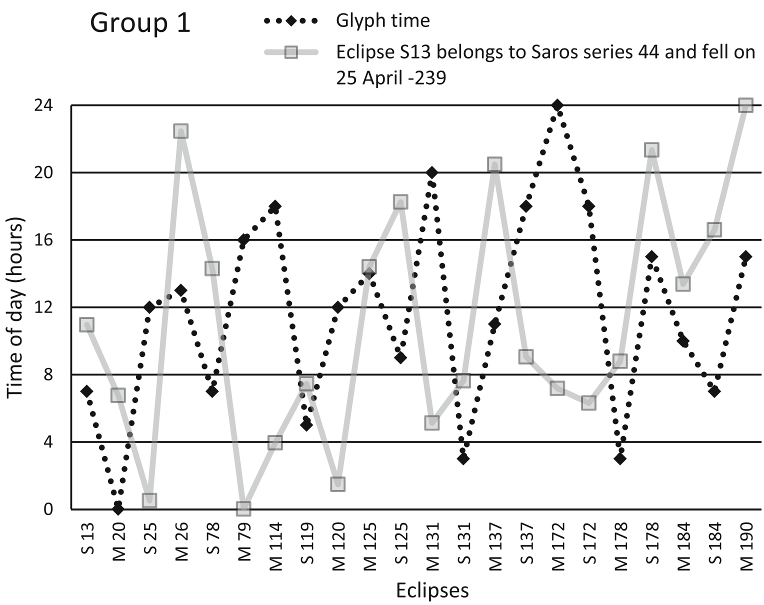


Fig. 12 Assuming that eclipse S13 corresponds to April 25, −239. (Opposition 1 corresponds to April 21, −240). The glyph times are out of phase with the actual times of day of the eclipses. The eclipses of group 1 in Table 14 are ruled out

The values of ΔT at the moment of each eclipse are also given by Espenak. The empirical values for AT of course come from the inscriptions on the Saros dial.

In Fig. 12, we plot the eclipse times from the Antikythera mechanism and the actual eclipse times for the Saros cycle beginning with the eclipse of −239 April 25, which we take as a representative of group 1. The two patterns are badly out of phase. Group 1 is therefore rejected.

In Fig. 13, we plot the eclipse times from the mechanism and the times for the Saros cycle beginning with the eclipse of −221 May 6, a representative of group 2. Group 2 is also rejected.

In Fig. 14, we plot the eclipse times from the Antikythera mechanism and the actual eclipse times for the Saros cycle beginning with the eclipse of −203 May 17, a representative of group 3. The fit is quite good in terms of phase, and so group 3 is confirmed. In Fig. 15, we repeat the graph of Fig. 14, but with the four outliers (13S, 120M, 125M, and 184M) excluded.

Conclusion of Sect. 16: *The eclipses of groups 1 and 2 in Table 14 are ruled out as candidates for the solar eclipse of month 13; group 3 is thus confirmed. The surviving candidates are therefore the eclipses in solar Saros 44 that lie in columns G, J, M, and P of Table* 10. *(These are the group three eclipses in Table* 14).

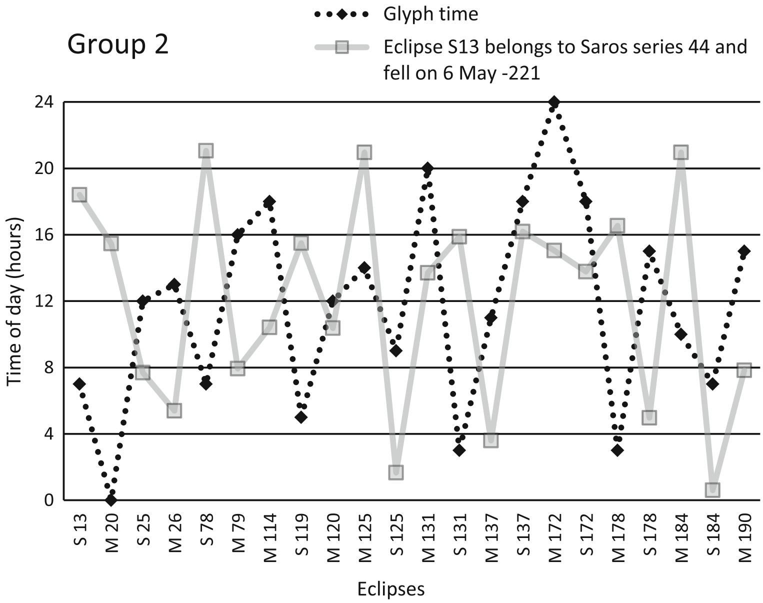


Fig. 13 Assuming that eclipse S13 corresponds to May 6, −221. (Opposition 1 corresponds to May 2, −222). The eclipses of group 2 in Table 14 are ruled out

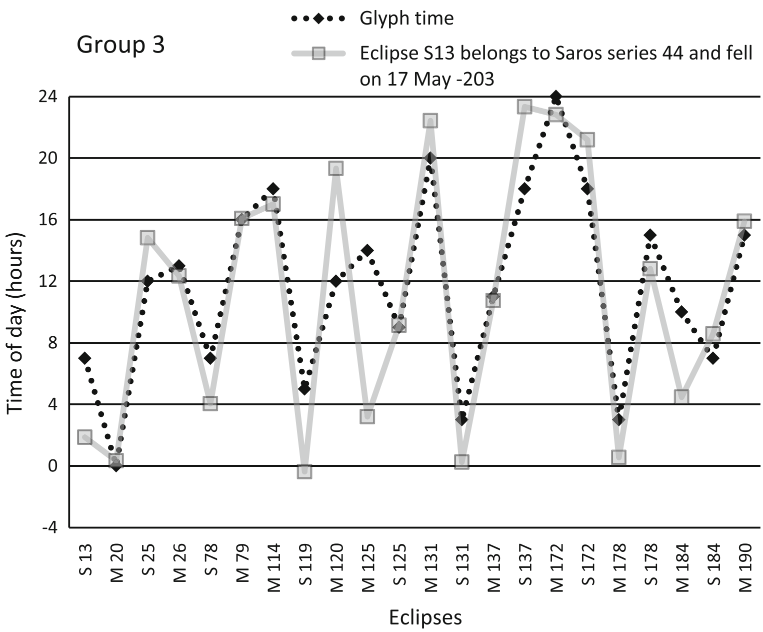


Fig. 14 Assuming that eclipse 13 corresponds to 17 May, −203. (The opposition of month 1 corresponds to May 12, −204). The fit in phase to the Antikythera mechanism is excellent. Solar eclipse 13 belongs to Saros series 44, Group 3

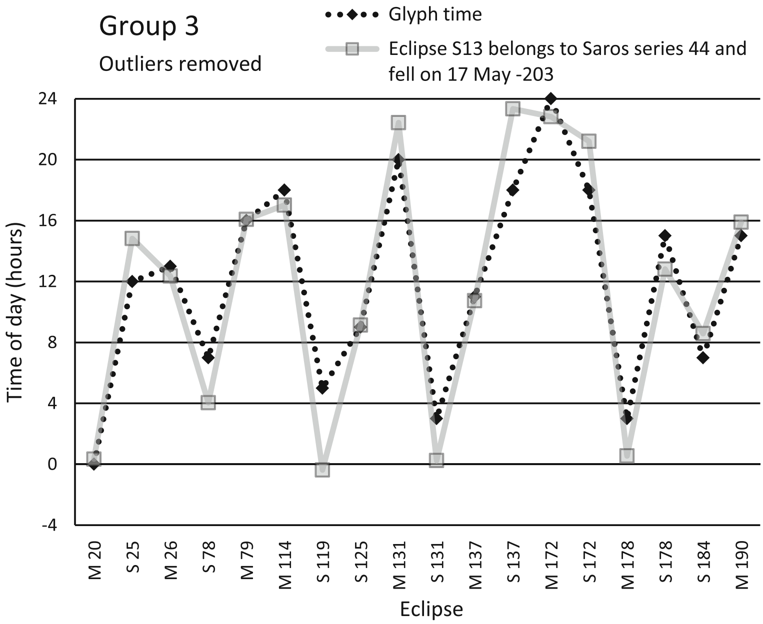


Fig. 15 Same as Fig. 14, with the four outliers excluded

17 Breakdown of the rule of 8 h

We have already alluded to the "rule of 8 h." If the time of eclipse is given for each eclipse in a Saros cycle (let us call it cycle 0), the approximate time for the corresponding eclipse in the next Saros cycle (cycle 1) can be obtained by adding 8 h. Adding 8 *more* hours gives the approximate eclipse time in cycle 2. And then in cycle 3 (since one complete *exeligmos* consisting of 3 Saros cycles has elapsed), the eclipse time returns approximately to its value in cycle 0.

However, the rule of 8 h is only approximately valid and it breaks down after a relatively small number of Saros cycles. In Fig. 16, we illustrate this breakdown using as example lunar Saros series 54. The lunar eclipses in the series come at successive intervals of 223 synodic months. We start from eclipse 0, the eclipse of −946 May 25, which fell at 23:02 UT according to the Espenak catalog.[16] (The last eclipse of this Saros series is for is 334 July 3). Plotted on the vertical axis is: the time of the eclipse predicted by the rule of 8 h *minus* the actual time of the eclipse. Thus for eclipse 1, we add 8 h to the time of eclipse 0 to obtain the "expected time" of eclipse 1 and then subtract the actual time of eclipse 1. For eclipse 2, we add 16 h to the time of eclipse 0 and subtract the actual time. For eclipse 3, we again expect the same time as for eclipse 0, from which we subtract the actual time of eclipse 3, and so on. As can be seen, the prediction for eclipse 1 is not bad—it comes only 1½ h from the expected

[16] Espenak actually gives the time of greatest eclipse in terms of TD. We have converted to UT.

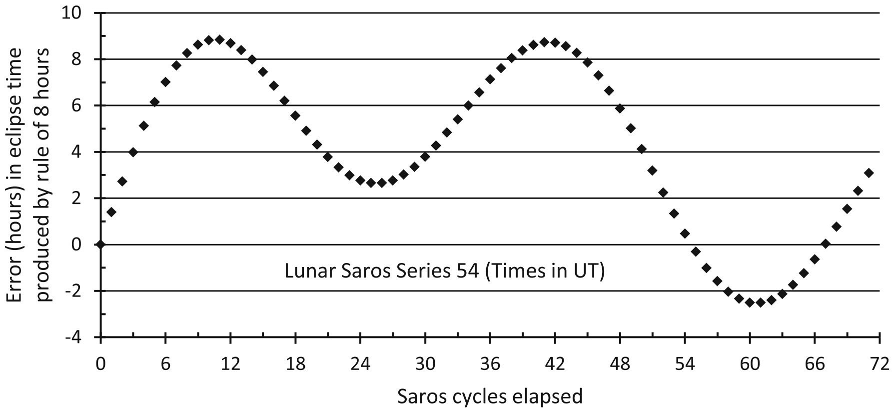


Fig. 16 Breakdown of the rule of 8 h illustrated for lunar Saros series 54. Successive data points are separated by 223 synodic months (one Saros cycle)

time. But when we have advanced by one whole *exeligmos* (eclipse 3), the expected time is more than 4 h off. After two exeligmos cycles, we are off by some 7 h. And the error produced by the rule of 8 h can be larger than 8 h! Moreover, each Saros series shows a different error pattern. Thus, if the eclipse predictor of the AM were designed to fit a particular Saros cycle reasonably well, it would fit other Saros cycles much less well. This provides a dating tool of considerable power.

In Table 14, each eclipse of group 3 is still a candidate. Each candidate is separated from its group 3 neighbors by an exeligmos cycle. Can we find a way to select one single eclipse of group 3 and exclude the others? We can, by exploiting the breakdown of the rule of 8 h.

For each eclipse of solar Saros series 44 in Table 10 (from about −329 to −77), we assume in turn that that eclipse was the eclipse of month 13, which began the 8-8- at the descending node. Then, for the eclipses of the Saros cycle that then starts, we compute the total squared error over a Saros (TSEOS). This we do by taking:

$$(\text{time of an eclipse from mechanism} - \text{actual time of same eclipse})^2$$

and summing over the extant eclipses of the Saros cycle. We use for the "actual time" the time of greatest eclipse in Espenak's catalogs of eclipses, for the eclipses that actually occurred. For eclipses predicted but not occurring, we use the time of true conjunction or true opposition, from Espenak's catalog of lunar phases. In variants of the procedure, we may sum only over the extant lunar eclipses, or only over the extant solar eclipses, or we may exclude outliers.

Figure 17 shows a TSEOS graph for solar Saros series 44. Thus, we assume, for each in turn, that the solar eclipse of month 13 is to be identified with one of the dates in Table 14, and we compare the mechanism's eclipse times with times deduced from modern theory. The quantity on the vertical axis is the sum of all the squared errors for the 18 eclipses. Now recall that, because of Figs. 12, 13, and 14, we already know that only one of three starting dates are still candidates—these are the dates of the eclipses of Group 3, and they are indicated in Fig. 17 by the solid black triangles. (The hollow

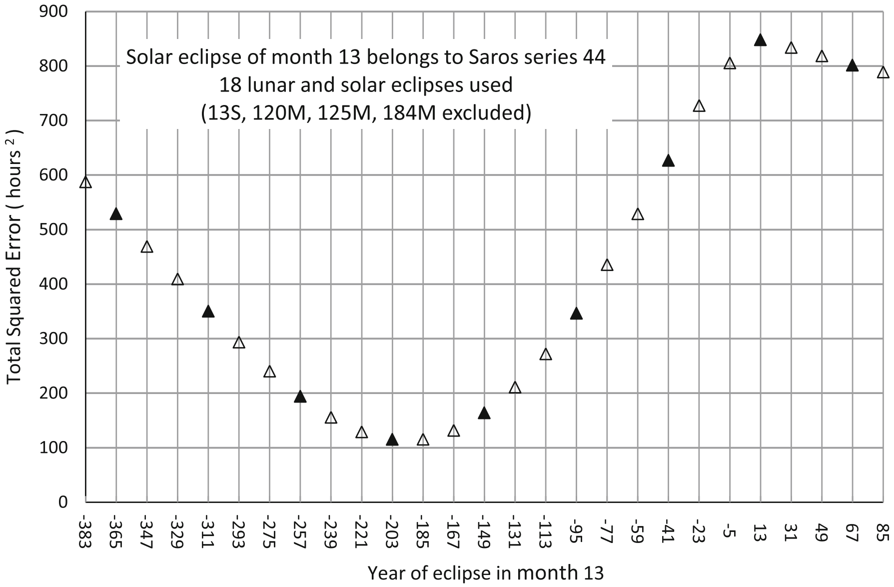


Fig. 17 Total squared error over a Saros (TSEOS) graph for solar Saros series 44. Eighteen eclipses are used, both solar and lunar. *Black triangles* indicate the solutions of Group 3 (in which the eclipse times of the mechanism are in the correct phase relation with the actual eclipses). Thus, the solar eclipse of month 13 most likely is to be identified with that of May, −203

triangles are already excluded). Thus, we conclude that the eclipse of month 13 is that of −203 May 17. And the full moon of month 1 corresponds to −204 May 12. The exeligmos dial read "0" at the time of this eclipse—i.e., no 8- or 16-h corrections are to be applied.

The deep, regular valley, with the well-defined minimum in Fig. 17 is again a confirmation that solar Saros 44 is the correct choice. For other Saros series, the minima are much shallower, and often there is no well-defined minimum at all. Examples of some of these other non-solutions are shown in Figs. 29 and 30.

In Fig. 17, we have used 18 eclipses, both lunar and solar. Using the nine lunar eclipses alone gives Fig. 18. From comparison of Figs. 17 and 18, it might be surmised that the timing errors in the solar eclipses are greater than in the lunar eclipses. That this is indeed the case is confirmed by Fig. 19.

Here are the RMS errors in the times of eclipse, assuming that indeed the solar eclipse of month 13 is that of −203 May 17:

9 lunar eclipses: 1.32 h
9 solar eclipses: 3.32 h
18 solar and lunar: 2.52 h

Clearly, the solar eclipse times are of lower quality. (And this is why, in Table 12, we included as options the bottom two rows, in which the solar eclipses are weighted by 50 %). Even if we imagine a different epoch for the solar eclipses and associate the solar eclipse of month 13 with that of −149, we would have an RMS timing error for the nine solar eclipses of 2.87 h—still double that of the lunar eclipses. In view of the

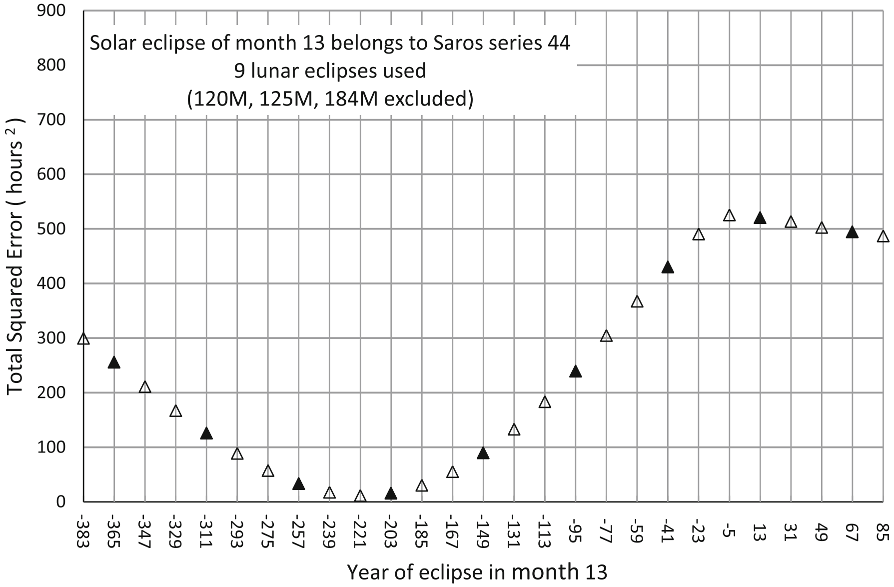


Fig. 18 TSEOS graph for solar Saros series 44, using the 9 lunar eclipses only

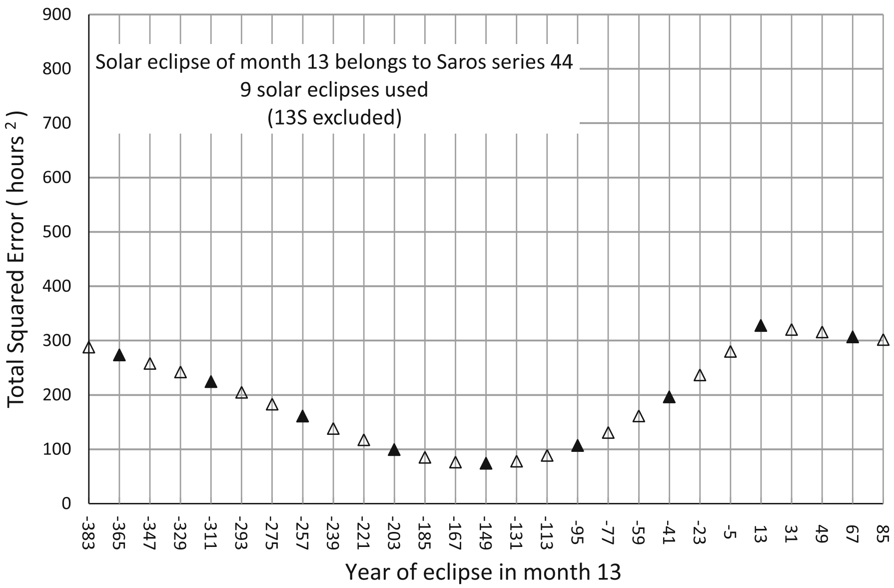


Fig. 19 TSEOS graph for solar eclipses only

lower quality of the solar eclipse times and the broad, flat bottom in Fig. 19, we do not believe that one can conclude that there were different epochs for the solar and lunar eclipses. However, this possibility cannot be completely excluded.

Steele (2000b, pp. 68–75) analyzes the accuracy of the predicted eclipse times surviving in Babylonian records. He distinguishes three categories for lunar eclipses: Category A, umbral lunar eclipses that were visible somewhere on the Earth's surface, but not necessarily at the longitude of Babylon; Category B, penumbral lunar eclipses, and Category F, failed predictions. He finds that the average error is for Category A 1.31 h and for Category B 2.86 h. Steele distinguishes two categories for solar eclipses: Category A, solar eclipses that were visible at Babylon or would have been visible there if the Sun were above the horizon at the time of the eclipse (that is, eclipses that were visible from the latitude, but not necessarily from the longitude of Babylon); and Category B, eclipses that would not be visible at Babylon under any circumstance. He found for Category A an error of 2.01 h and for Category B an error of 3.55 h.

The accuracy of the times of the lunar eclipses on the AM is comparable to that of Babylonian lunar eclipse predictions of category A. This is presumptive evidence that the lunar eclipse times on the AM are the results of some kind of predictive scheme—and not directly of observation. (We will show this more explicitly in Sect. 19).

We have also calculated the mean timing error for the 18 eclipses (with outliers excluded). The result is −0.11 h, i.e., very close to zero.[17] This suggests that we are on the right track by modeling the ancient mechanic's procedure by starting with the mean time of the eclipse and adding corrections. This also suggests that the eclipse times recorded on the mechanism's Saros dial are indeed intended to represent the times of mid-eclipse, which confirms a conjecture of Alexander Jones (personal communication). Babylonian practice seems to have favored prediction of the onset of the eclipse (Steele 2000b, p. 73). So here is another example of a recasting or adaptation of Babylonian conventions by the maker of the Antikythera mechanism.

Conclusion of Sect. 17: The solar eclipse of month 13 corresponds to that of −203 May 17. The full moon of month 1 was that of −204 May 12.

18 Analysis of the underlying lunar and solar models

In our attack on the dating problem, we have worked with sinusoidal forms for the equations of center of the Moon and Sun because these are simple to handle analytically and are more than good enough for the problem at hand. But the question does naturally arise as to what sort of theory underlies the eclipse times on the Antikythera mechanism. Geminus, a writer of the first century BCE, describes for Greek readers in his *Introduction to the Phenomena* (xviii 4–19) the essential features of the Babylonian lunar theory now known as System B (Evans and Berggren 2006, pp. 96–99, 228–230). The Moon's daily motion changes from day to day according to a simple saw-tooth pattern, with uniform daily changes of 0;18° between maximum and minimum daily displacements of 15; 14, 35° and 11; 6, 35°. (We use the Neugebauer notation, in which whole degrees stand to the left of the semi-colon, and successive sexagesimal parts stand to the right and are themselves separated by commas). This

[17] Using all 22 eclipses, we get for the mean timing error −0.73 h.

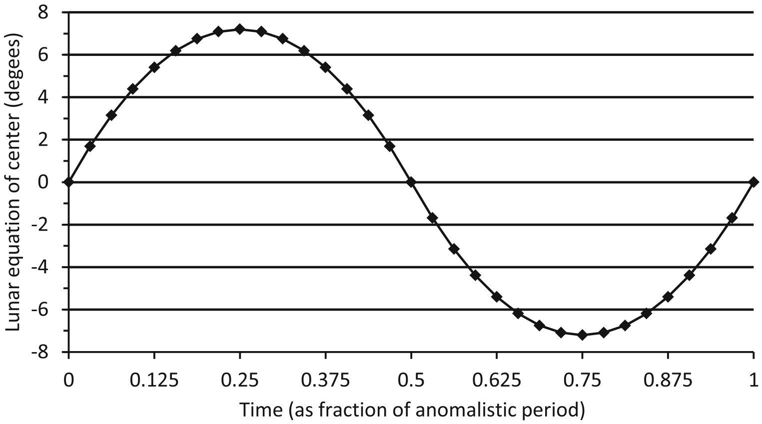


Fig. 20 The effective equation of center resulting from the Babylonian lunar theory of System B. The *left* and *right* halves of the curve are parts of parabolas. In the *graph*, time is reckoned from the effective perigee

leads to a lunar "equation of center" of quadratic form:

$$q = DT\left[\frac{1}{2}\frac{t}{T} - \left(\frac{t}{T}\right)^2\right], \quad 0 \le t \le T/2$$

$$q = DT\left[-\frac{1}{2}\left(\frac{t}{T} - \frac{1}{2}\right) + \left(\frac{t}{T} - \frac{1}{2}\right)^2\right], \quad T/2 \le t \le T.$$

q (the Moon's equation of center) is the difference between the Moon's longitude according to System B and the longitude it would have if it moved uniformly. The time t is reckoned from the moment of fastest motion ("perigee"), T is the length of the anomalistic month (27; 33, 20 days)[18], and D is the difference between the greatest and least daily motion (in degrees per day). The curves for q are segments of parabolas, alternately concave downward and concave upward, as shown in Fig. 20. The amplitude is plotted according to the parameters quoted by Geminus, which appear also in the Babylonian material (Neugebauer 1975, p. 480). A function like this one (with a slightly different amplitude) would model the eclipse times on the Antikythera mechanism about as well as would the sinusoids we have used for analysis.

For the solar theory, the simplest Babylonian approach belongs to System A. Here, the Sun moves at a uniform angular speed of 30° per synodic month in the fast zone of the zodiac (which stretches from Virgo 13° to Pisces 27°) and 28 1/8° per synodic month in the slow zone. The resulting equation of center is piecewise linear, as shown in Fig. 21. How well would these two theories match the eclipse data from the AM?

[18] But in the analysis in this paper, we use the length of the anomalistic month that is implicit in the Antikythera mechanism, which is (223/239) times the synodic month. For the synodic month, we use $365.25^{\mathrm{d}}(19/235)$. This makes no practical difference: An anomalistic month of 27.556 days for System B and Geminus, 27.554 days implicit in the AM.

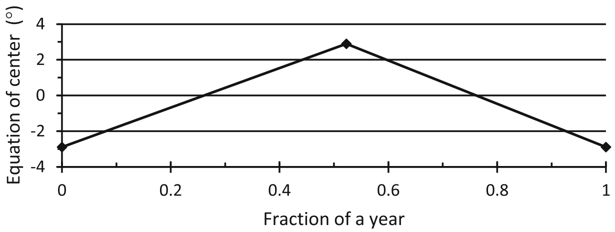


Fig. 21 The effective equation of center resulting from the Babylonian solar theory of System A. In the *graph*, time is reckoned from the beginning of the fast zone. The zero crossings are the effective perigee (*left*) and apogee (*right*)

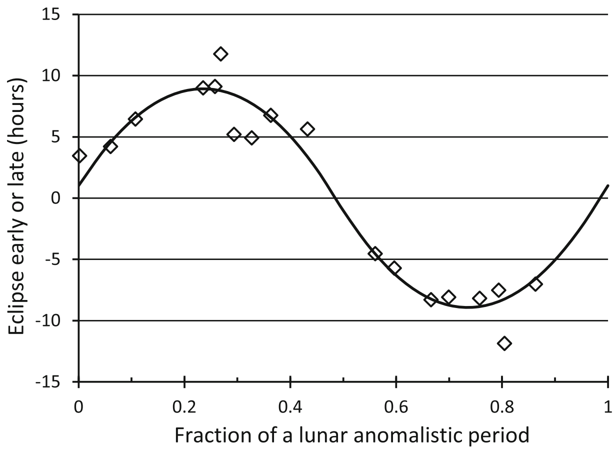


Fig. 22 System B style lunar theory (*black curve*) fitted to the eclipse times of the Antikythera mechanism with the solar fit removed (*open diamonds*). $t = 0$ is the opposition of month 1. Eighteen lunar and solar eclipses are used

We have fitted these Babylonian-style functions to the Antikythera mechanism eclipse times, using 18 eclipses (9 solar and 9 lunar, with the four outliers removed).[19] We require the width of the fast zone to be 194° and that of the slow zone 166°, as in the Babylonian solar theory of System A. The adjustable parameters are the amplitudes of the solar and lunar equations of center, their phases, and $\bar{h}_0$—thus, five parameters as with the sinusoidal fits. The results are shown in Figs. 22 and 23. The parameters of the fit are as follows:

Moon amplitude of equation of center curve: 8.88 h
"mean anomaly"[20] at opposition of month 1: 185.0°

[19] Our technique is the same as that used in Sect. 13. We compute the lunar and solar equations of center for the given models and assume that these need to be compensated for by the Moon and Sun running at their mean relative motion of 12.2°/day. Once we know the locations of the perigees, it is possible to apply a more sophisticated reckoning, in which we take into account the variable speeds of the Moon and Sun. This is done in Sect. 19.

[20] The mean anomaly does not have a clear-cut definition as an angle in this non-geometrical model. The parameter determined from the fit is the offset of the zero crossing from $t = 0$ in Fig. 22, which is 0.01392

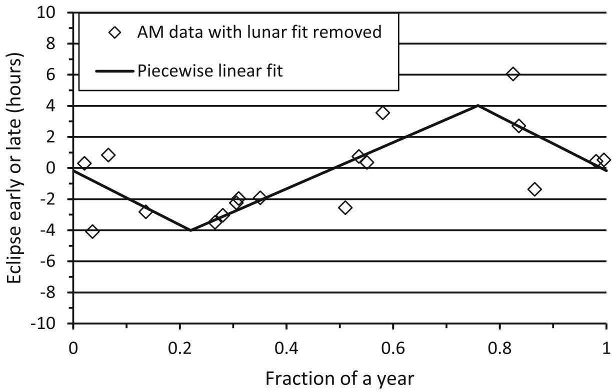


Fig. 23 Babylonian-style, piecewise-linear solar equation (*black line*) fitted to the AM data with the lunar fit removed (*open diamonds*). $t = 0$ is the opposition of month 1. The widths of the fast and slow zones are as in the Babylonian solar theory of System A. The amplitude and the place of the left-hand zero crossing were adjustable parameters for the fit

Sun amplitude of sawtooth function: 4.02 h
"Mean anomaly"[21] at opposition of month 1: 183.7°
$\bar{h}_0$ -9.39 h.

These are all comparable to the values shown in Table 12 for the sinusoidal fits. Visually, the quality of the Babylonian fit is good—and statistically this is borne out as well. The parabolic (for the Moon) and piecewise-linear (for the Sun) fits leave a sum of squared residuals of 53.4 h^2. This is defined as: (glyph time *minus* time predicted by fit)^2, summed over the 18 eclipses. For the sinusoidal fit, using the same 18 eclipses, the residuals come to 57.0 h^2. The difference is due mostly to the better performance of the piecewise-linear solar theory. The difference in performance is not large enough to prove the use of Babylonian-style theories for handling the non-uniformity of motion, but this does seem a plausible conjecture, especially as there is evidence of the use of Babylonian solar theory of System A on the front side of the mechanism, in the differential graduation of the zodiac and Egyptian calendar scales

Footnote 20 continued
of an anomalistic period. Note that at $t = 0$, the eclipses are slightly late and getting later; thus, the equation of center is slightly negative and getting more negative, i.e., we are near (and just after) apogee, by $0.01392 \times 360° = 5.0°$, hence the value 185.0°.

21 The mean anomaly does not have a clear-cut definition as an angle in this non-geometrical model. The parameter determined directly from the fit is the t-value of the most negative point in Fig. 23, which is 0.24084 of a year. The zero crossings of the graph are the moments when the equation of center is zero, which we associate with "perigee" and "apogee." As can be seen, at $t = 0$, the eclipses are very slightly early, and getting earlier. Hence, we are near, and just after, apogee. The zero crossing would have occurred at $t = -0.01028$ year. Hence, we take the mean anomaly to be $0.01028 \times 360°$ beyond apogee, i.e., 183.7°. Note that in this Babylonian-inspired model, the maximum and minima are not uniformly spaced. However, the zero crossings are.

(Evans et al. 2010). Moreover, given our likely epoch of −204, it is highly unlikely that trigonometric equations of center would have been available.

We also tried fitting the eclipse times with an epicycle theory, for both the Sun and the Moon. In the case of an epicycle theory, the equation q of center can be found from the following relation:

$$\sin q = \frac{e \sin \bar{\alpha}}{\sqrt{1 + e^2 - 2e \cos \bar{\alpha}}},$$

where $\bar{\alpha}$ is the mean anomaly, measured from the perigee, and e is the ratio of the epicycle's radius to the radius of the deferent circle. In this case, the sum of the squared residuals turns out to be 61.4 h^2. So the classic epicycle, characteristic of late Greek astronomy, does less well than simple sinusoidal fits. The reason seems to be the no longer 180° spacings between positive and negative peaks of q. As can be seen from Fig. 22, the 180°-spacings of the positive and negative peaks in the Babylonian lunar theory fit the eclipse times well. But again the difference is not great enough to allow us to reject the epicycle theory on the basis of the numbers alone.

In fitting the piecewise-linear theory to the eclipse times, we also tried relaxing the Babylonian condition that the Sun's fast zone should be 194° wide and the slow zone 166°; thus, in a second trial, we left the relative size of these widths as a free parameter. We obtained: 205° and 155°, with, indeed, the fast zone broader. (This is reassuring, as it could have turned out the other way). The fits in this case are as follows:

Moon amplitude of equation of center curve: 8.92 h
"mean anomaly" at opposition of month 1: 185.3°
Sun amplitude of sawtooth function: 3.99 h
"mean anomaly" at opposition of month 1: 183.1°
$\bar{h}_0$ −9.44 h

with residuals of 53.2 h^2, negligibly smaller than for the zones of Babylonian width.

The apparently poorer performance of the solar theory in all the fits (both sinusoidal and Babylonian) could indicate a deficiency in our method of converting the eclipse times of the inscriptions to modern 24-h time, or the use by the ancient mechanic of some tool other than epicycles or saw-tooth functions for getting the eclipse times of the Sun. Fortunately, this does not affect the dating problem in a significant way. The excellence of the lunar fit (Figs. 7 or 22) does suggest that we are on the right track in supposing that the eclipse times were obtained by using the Babylonian eclipse records to obtain the day and mean time and then applying an "equation of center" to correct the time. Even if Babylonian theories were used, as seems likely, the equation of center would presumably have to be adapted. For example, the maximum equation of center for Moon in system B is, as we can see in Fig. 20, about 7°, which translates into an advance or delay in eclipse time of nearly 14 h, instead of the more realistic value of about 10 h reflected by the Antikythera mechanism.

Conclusion of Sect. 18: The eclipse times on the Saros dial would be somewhat better fitted by equations of center based on Babylonian-style arithmetical functions than by epicycles. The improvement in performance by the Babylonian theories is not great enough to prove that they were actually used. But, given the early likely epoch date of the eclipse predictor, it is highly unlikely that a trigonometric treatment based on epicycles was used.

19 Synthesis of eclipse times

Geometers from antiquity to the seventeenth century often maintained that a complete solution of a problem required two parts, characterized by different methods—analysis and synthesis. In Sect. 18, we applied analytical methods to the problem of the eclipse times to uncover information about the underlying model. We found that Babylonian-style arithmetical theories for the equations of center of the Moon and Sun would work somewhat better (in the sense of agreeing better with the inscriptions) than would an epicycle theory. Because of the early date of the likely epoch of the Saros dial, we have good reason to doubt that trigonometric functions or epicycles could have been involved. Moreover, we found in Sect. 13 that the lunar mean anomaly and the solar mean anomaly were near 180° (at apogee) at the full Moon of month 1. These results leave the model with little wiggle room.

Thus, we adopt a lunar theory like that of Babylonian system B. The equation of center was given in Sect. 18. The excess lunar velocity v_E in this theory can be found by differentiation with respect to t:

$$v_E = \frac{\mathrm{d}q}{\mathrm{d}t} = D\left[\frac{1}{2} - \frac{2t}{T}\right], \quad 0 \le t \le T/2$$
$$v_E = \frac{\mathrm{d}q}{\mathrm{d}t} = D\left[-\frac{3}{2} + \frac{2t}{T}\right], \quad T/2 \le t \le T.$$

(Ancient astronomers would have done it differently, but had the means to go back-and-forth between daily velocity and accumulated distance). Recall that D is the difference between the maximum and minimum values of the daily velocity (in degrees per day), which is a characteristically Babylonian parameter. t is reckoned in days from perigee, and T is the length of the anomalistic month. The total angular speed of the Moon (in degrees per day) is

$$v = \bar{v} + v_E,$$

where $\bar{v}$ is the Moon's mean daily motion = mean speed of Moon w/r Sun + mean speed of Sun. Thus $\bar{v} = 360°/\mathrm{sm} + 360(19/235)°/\mathrm{sm} = 360(1 + 19/235)°/\mathrm{sm}$. Since 1 sm = $(19/235)365.25^{\mathrm{d}}$, we obtain for the mean daily motion of the Moon

$$\bar{v} = (360/365.25)(235/19 + 1)°/\mathrm{day}.$$

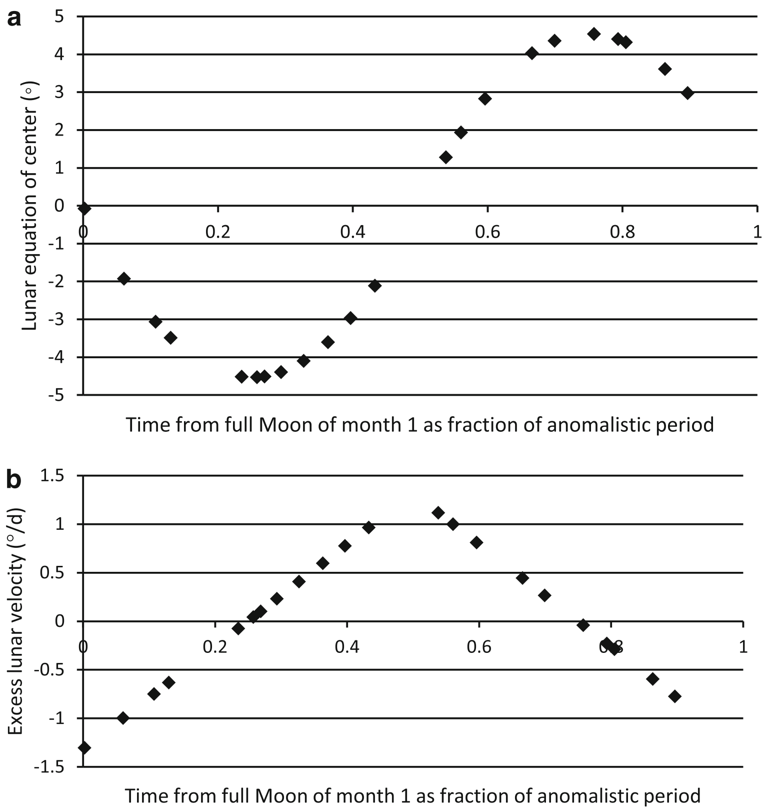


Fig. 24 **a** A Babylonian-style lunar theory for the Saros dial of the Antikythera mechanism: equation of center. The plotted points give the theoretical value of the Moon's equation of center for the eclipses on the Antikythera mechanism. **b** A Babylonian-style lunar theory for the Saros dial of the Antikythera mechanism: excess velocity. The plotted points give the theoretical value of the Moon's excess velocity (above mean motion) for the eclipses on the Antikythera mechanism

Because of the evidence mentioned above, we start with the Moon at apogee at the full Moon of month 1. Thus, the theoretical graphs of q and v_E look as shown in Fig. 24a, b. Note that there is now only one adjustable parameter for the pair of them, namely D.

For the Sun, the graphs look like Fig. 25a, b. We start with the Sun at apogee at the full Moon of month 1. Now, we must do a little figuring to cast the theory into a one-parameter model consistent with the overall structure of the Babylonian solar theory. Let V_S and V_F denote the Sun's slow and fast speeds in the two-zone theory, expressed for the time being in degrees per mean synodic month (°/sm). The fast zone has a width of 194° and the slow zone 166°. Thus, the number of synodic months spent in the fast zone is $N_\mathrm{F} = 194/V_\mathrm{F}$, while the number of months in the slow zone is $N_\mathrm{S} = 166/V_\mathrm{S}$. The sum of these must equal the number of synodic months in a year:

$$\frac{194}{V_\mathrm{F}} + \frac{166}{V_\mathrm{S}} = \frac{235}{19}.$$

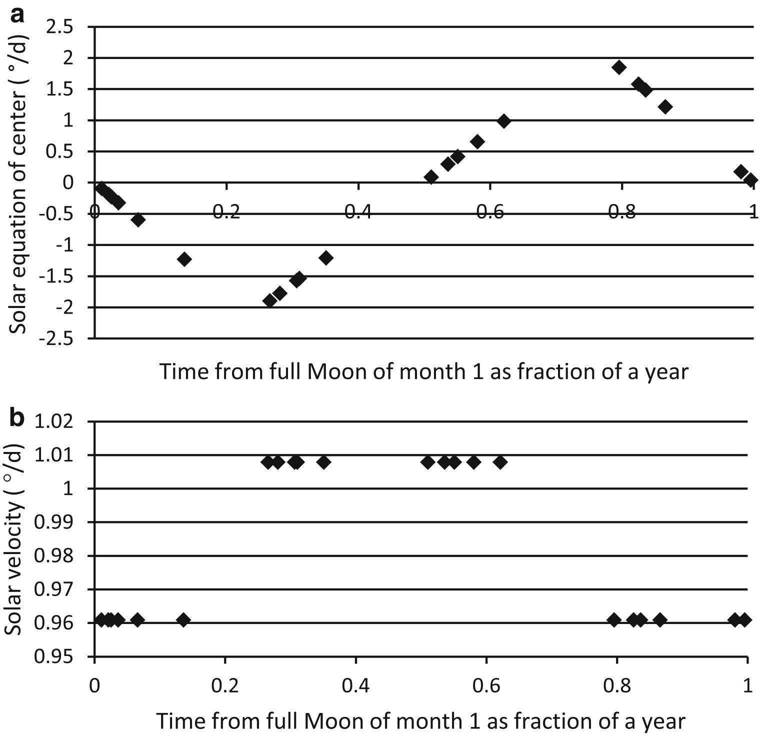


Fig. 25 **a** A Babylonian-style solar theory for the Antikythera mechanism: equation of center. The points plotted correspond to the eclipses on the Antikythera mechanism. **b** A Babylonian-style solar theory for the Saros dial of the Antikythera mechanism: solar velocity

Thus, if we treat V_F as a free parameter, V_S is not independent:

$$V_S = \frac{166}{\frac{235}{19} - \frac{194}{V_F}}.$$

Finally, we must calculate the time values for the ends of the straight-line segments in Fig. 25a. The upward-sloping segment is in the fast zone. The time A_F spent in the fast zone is $194/V_F$ s, or

$$A_F = \left(\frac{194}{V_F}\right) \frac{19 \times 365.25}{235} \text{ days.}$$

The number of days A_S spent in the slow zone is $A_S = 365.25 - A_F$. Thus, A_S and A_F both depend on the value of the parameter V_F. In Fig. 25a, the lead-off downward-sloping section is in the slow zone and ends at the value $t = A_S/2$. The long rising section ends at the value $t = A_S/2 + A_F$. With these relations, the solar theory is now expressed in terms of a single free parameter, V_F.

We may now easily calculate the predicted times of the eclipses according to this scheme. We assume that the mean time of the eclipse is equal to the local clock time of the opposition of month 1, plus the advance in clock time that would result from the mean motions alone, plus a term $-(q_{☾} - q_{\odot})/(v_{☾} - v_{\odot})$, where each q is expressed in degrees and each v in degrees per day. Note that at the time of a given eclipse, $v_{\odot}$ is just either one of two constant values, V_{F} or V_{S}. For the Moon, $v_{☾}$ is calculated from the formulas given above, i.e., $v_{☾} = \bar{v}_{☾} + v_{E☾}$.

Explanation: The full Moon of month 1 occurred at a certain time of day t_1; both $q_{☾}$ and $q_{\odot}$ were then zero. At some later eclipse, if $q_{☾} - q_{\odot}$ is again zero, the time of the eclipse will be greater than t_1 owing to the effects of the mean motions alone (as explained in Sect. 13), thus producing an eclipse at a certain time of day t_2. But if $q_{☾} - q_{\odot} > 0$, the eclipse will occur earlier than t_2. If $q_{☾} - q_{\odot} < 0$, the eclipse will occur later than t_2. Rather than dividing the difference in the equations of center by the constant mean relative velocity of the Moon and Sun $(\bar{v}_{☾} - \bar{v}_{\odot})$, we divide by the relative velocity $(v_{☾} - v_{\odot})$ that actually holds at the moment of the mean eclipse. (Thus, in contrast to the procedure in the earlier, preliminary investigation, we are explicitly taking into account the variable speeds. This is now easy to do, since we now know the initial values of the mean anomalies).

In Table 15, for each eclipse in column (1), we have its time of day as given by the Saros dial but expressed in modern terms (2). Column (5) gives the advance in time of the eclipse, from the time of opposition 1, owing to the effects of mean motions alone (calculated as explained in Sect. 13). Columns (3) and (4) give the position of the mean eclipse in the anomalistic period or in the year, respectively, reckoned from the opposition of month 1. Columns (6)–(9) give $q_{☾}$, $q_{\odot}$, $v_{☾}$, and $v_{\odot}$, all calculated from the Babylonian-style theories, by the methods just explained. Then, the predicted time of day of an eclipse (PTD) is given in column (10) by

$$\mathrm{PTD} = \text{time of opposition } 1 + (5) - \frac{(6)-(7)}{(8)-(9)} \times 24,$$

expressed in hours (local time).

We compute the sum of the squared residuals (SSR) by taking [column (10)–column (2)]^2 and summing over all 18 eclipses. We minimize this sum with respect to the three free parameters, D, V_{F} and $\bar{h}_0$ (the time of day of the opposition of month 1). The results are shown in Figs. 24a, b, 25a, b. Thus, one may read off the values of $q_{☾}$, $q_{\odot}$, $v_{☾}$, and $v_{\odot}$ on these graphs and compare with the values given in Table 15. The results of the fits for the amplitudes of the equations of center are comparable to what we have seen before. The parameters of the fit are: $\bar{h}_0 = -9.40\,\mathrm{h}$, $D = 2.634°/\mathrm{day}$, and $V_{\mathrm{F}} = 29.762°/\mathrm{sm}$, all quite reasonable. The value of D corresponds to an amplitude of 4.54° for the lunar equation of center. The value of V_{F} corresponds to an amplitude of 2.14° for the sawtooth that is the solar equation of center function. These are in about the right range to function well in terms of actual phenomena. But they are a bit smaller than the corresponding values from standard Babylonian theory—another sign of adaptation by the designer of the Antikythera mechanism.

As can be seen by comparing columns (10) and (2), or by examining Fig. 26, this scheme does quite well at matching the inscribed eclipse times. The agreement is compelling enough to give some hope that we are on the right track. Of course, some of the timing errors are larger than one would like to see (up to 3½ h). However, the

Table 15 Prediction of eclipse times using Babylonian-style equations of center and velocities

1 Eclipse or opp.	2 Time of day from AM (h)	3 Fraction of anomalistic month	4 Fraction of year	5 Advance due to mean motions (h)	6 $q_{☾}$ (°)	7 $q_{\odot}$ (°)	8 $v_{☾}$ (°/d)	9 $v_{\odot}$ (°/d)	10 Predicted time of day (h)
1 op	−9.40	0	0	0	0	0	11.86	0.96	14.60
20 M	0	0.363229	0.53617	2.07	−3.61	0.29	13.78	1.01	24.00
25 S	12	0.257848	0.980851	12.14	−4.53	0.17	13.22	0.96	11.95
26 M	13	0.793722	0.021277	6.51	4.40	−0.19	12.95	0.96	11.92
78 S	7	0.060538	0.265957	15.38	−1.93	−1.90	12.18	1.01	6.05
79 M	16	0.596413	0.306383	9.75	2.82	−1.57	13.99	1.01	16.22
114 M	18	0.107623	0.13617	23.67	−3.07	−1.23	12.43	0.96	18.11
119 S	5	0.002242	0.580851	9.74	−0.08	0.66	11.87	1.01	1.97
125 S	9	0.432735	0.065957	14.18	−2.11	−0.60	14.14	0.96	7.54
131 M	20	0.327354	0.510638	0.26	−4.10	0.09	13.58	1.01	22.85
131 S	3	0.863229	0.551064	18.63	3.61	0.41	12.58	1.01	2.61
137 M	11	0.757848	0.995745	4.70	4.53	0.04	13.13	0.96	10.44
137 S	18	0.293722	0.03617	23.07	−4.40	−0.33	13.41	0.96	21.52
172 M	24	0.269058	0.825532	18.61	−4.51	1.58	13.28	0.96	21.08
172 S	18	0.804933	0.865957	12.98	4.32	1.21	12.89	0.96	21.33
178 M	3	0.699552	0.310638	23.06	4.35	−1.54	13.44	1.01	2.29
178 S	15	0.235426	0.351064	17.43	−4.52	−1.21	13.10	1.01	14.61
184 S	7	0.665919	0.83617	21.87	4.02	1.48	13.62	0.96	7.65
190 M	15	0.560538	0.280851	7.94	1.93	−1.78	14.17	1.01	15.78

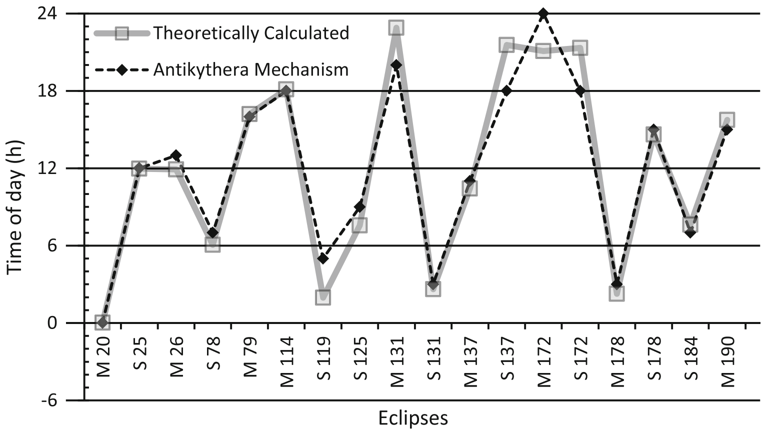


Fig. 26 Comparison of an eclipse prediction scheme based on Babylonian-style lunar and solar theories with the eclipse times given by the Antikythera mechanism

times are given on the AM in terms of whole hours. Moreover, we have computed equations of center and lunar velocities for particular moments in time, while the ancient mechanic probably would have had a table of discrete values. There is also the possibility of a deficiency in our conversion of AM times to standard clock times, as discussed in Sect. 13. A curious feature is the fact that the five eclipses with timing errors near 3 h all took place either a few hours before or a few hours after midnight. So there could be a source of systematic error, either in our understanding of the ancient mechanic's procedure, or in the procedure of the ancient mechanic himself. Nevertheless, the performance of the Babylonian-inspired scheme is quite good: It has a root-mean-square discrepancy with the AM inscriptions of only 1.76 h. The RMS discrepancies for the lunar and solar eclipses are of comparable size, 1.47 and 2.01 h, respectively, suggesting that similar methods were used to compute the lunar and solar eclipses. The minor difference in RMS discrepancies is probably not significant, given that we excluded more lunar than solar eclipses among the outliers.[22]

We have also tried an epicycle theory—in which the variable motions of both the Sun and the Moon are represented by epicycles. Again, we take both the solar and lunar mean anomaly to be 180° at the full Moon of month 1. We compute predicted times exactly according to the scheme described in this section, the only difference being the forms of the equation of center functions. So, again there are just three parameters to be varied—the radii of the solar and lunar epicycles and $\bar{h}_0$. We fit the parameters by requiring the sum of the squared residuals (SSR) in the eclipse times to be a minimum. The results are $r_{\odot} = 0.0304$, $r_{☾} = 0.0851$, $\bar{h}_0 = -9.31$ h, where the radii of the deferent circles are taken to be unity.[23] These parameters result in a graph that differs only very subtly from Fig. 26. Again, the epicycles (sum of squared residuals = 58.0 h^2) fit the inscriptions only a little less well than does the Babylonian-style theory

[22] The Babylonian-style method described here is actually a pretty good predictor of eclipse times: Its RMS discrepancy with the true eclipse times is 1.51 h.

[23] The maximum values of the equations of center corresponding to these epicycle radii are about 1.75° for the Sun and 4.90° for the Moon.

($55.8\,h^2$). Thus, while a correction to the mean times based on equations of center and velocities appears quite plausible, it is not possible to reject the epicycles solely on the basis of the goodness of fit.

Finally, we have explored the question of whether daily velocities were really used, or just the mean speeds (which we used in our preliminary analysis in Sect. 13). The use of daily velocities does improve the fit between the theory and the AM times, but not by enough to be decisive. For the Babylonian-style theory, the SSR (sum of squared residuals) is $55.8\,h^2$ using daily velocities, but 58.1 using the mean speeds only. For the epicycle theory, the SSR is $58.0\,h^2$using daily velocities but 59.8 using only the mean speeds.

Conclusion of Sect. 19: An eclipse prediction scheme in which the months of the eclipses are taken from a Babylonian list (or 8-8-7-8-7- scheme) and the times of the eclipses are reckoned using equations of center and velocities based on Babylonian-style solar and lunar theories fits the eclipse times on the Antikythera mechanism well. Epicycle theories for the non-uniform motion of the Sun and Moon fit the AM eclipse times slightly less well, but cannot be excluded on this basis alone.

20 Longitude of best fit

We did not need a very precise value of the terrestrial longitude in order to identify the sequence of eclipses on the AM. No longitude at all was required in order to place the eclipse of month 13 in solar Saros cycle 44—this was done by means of the lunar anomaly. Only a rough value of the longitude was needed for ruling out Groups 1 and 2. Also, the TSEOS graphs that were used to identify a single most likely candidate among the survivors of Group 3 required only a rough value of the longitude. But now that the eclipses on the Saros dial are identified with particular actual eclipses, it is possible to perform a test to establish a longitude of best fit and to verify that it is consistent with our previous assumption. Of course, the great body of the data incorporated on the Saros dial does not come directly from observation: The months of the eclipses come from the 8-8-7-8-7- scheme in current use, and the times of day for the individual eclipses were calculated from a theory. However, at root there was some sort of connection with observation, perhaps based on a small number of eclipses. This is what, after all, necessitated the occasional Babylonian recalibrations. And thus, it does make sense to enquire for which range of terrestrial longitudes the eclipse times on the AM would be in fair concordance with the phenomena.

In Table 16, the two leftmost columns contain essential information from the Antikythera mechanism: the months and types of the eclipses, and their times of day (converted into modern 24-h time as explained in Sect. 13). The next four columns contain information from Espenak's catalog: lunation number, actual date, and actual time of the eclipse, as well as the value of ΔT. The identification of the AM eclipses with particular eclipses in Espenak depends, of course, on our demonstration in Sect. 17 that the solar eclipse of month 13 is that of −203 May 17. In the

Table 16 Comparison of eclipse times on the Antikythera mechanism with the actual times of the eclipses in UT, for deducing the longitude of best fit

Eclipse	AM time (h)	Lunation number	Date (Espenak)	TD (Espenak)	ΔT (s) (Espenak)	UT (h)	AM–UT (h)
20 M	0	−27237	−203 Nov 25	2:18:04	12758	−1.24	1.24
26 M	13	−27231	−202 May 20	14:18:45	12752	10.77	2.23
79 M	16	−27178	−198 Sep 1	18:01:27	12700	14.50	1.50
114 M	18	−27143	−195 Jul 1	18:56:57	12666	15.43	2.57
131 M	20	−27126	−194 Nov 16	0:22:07	12649	20.86	−0.86
137 M	11	−27120	−193 May 11	12:40:18	12643	9.16	1.84
172 M	24	−27085	−190 Mar 11	0:44:45	12609	21.24	2.76
178 M	3	−27079	−190 Sep 3	2:28:11	12604	−1.03	4.03
190 M	15	−27067	−189 Aug 23	17:49:16	12592	14.32	0.68
25 S	12	−27231	−202 May 6	16:47:22	12752	13.25	−1.25
78 S	7	−27178	−198 Aug 18	5:59:42	12700	2.47	4.53
119 S	5	−27137	−195 Dec 11	1:33:19	12660	−1.96	6.96
125 S	9	−27131	−194 Jun 6	11:04:05	12655	7.55	1.45
131 S	3	−27125	−194 Nov 30	2:10:48	12649	−1.33	4.33
137 S	18	−27119	−193 May 27	1:16:17	12643	21.76	−3.76
172 S	18	−27084	−190 Mar 24	23:07:08	12609	19.62	−1.62
178 S	15	−27078	−190 Sep 18	14:44:22	12603	11.24	3.76
184 S	7	−27072	−189 Mar 14	10:29:38	12597	6.99	0.01

7th column, we have expressed Espenak's times of greatest eclipse in terms of UT rather than TD. These times in UT are to be compared with the times in column 2: Antikythera time (AT) $=$ UT $+ \Delta L$, where ΔL is the longitude of best fit, in hours east of Greenwich. Thus, we form the quantity

$$\sum_{i=1}^{18} (\mathrm{AT}_i - \mathrm{UT}_i - \Delta L)^2,$$

And minimize it by varying ΔL, with the result

$$\Delta L = 1.69\,\mathrm{h} = 25.3^\circ.$$

Thus, the longitude of best fit is $1^{\mathrm{h}}41^{\mathrm{m}}$ east of Greenwich, which corresponds to the eastern Aegean—an unsurprising result. The confidence intervals are:

$$\begin{aligned} \pm 0.58^{\mathrm{h}} &= \pm 8.7^\circ \quad \text{for } 67\,\% \\ \pm 0.98^{\mathrm{h}} &= \pm 14.7^\circ \quad \text{for } 90\,\% \\ \pm 1.17^{\mathrm{h}} &= \pm 17.5^\circ \quad \text{for } 95\,\%. \end{aligned}$$

In words, any place within about an hour of longitude from the longitude of best fit would do well enough. Thus, no part of the Greek world of the eastern Mediterranean may be safely ruled out on the basis of the eclipse times alone. The 95 % confidence interval takes us from 7.8°E longitude to 42.8°. However, we note that Babylon (longitude 44.5°E) is outside this generously wide zone. Thus, it seems likely that the calibration of the eclipse times on the AM included a Greek component, either a locally observed eclipse time or, perhaps more likely, the subtraction of an hour or so from a Babylonian calibration.

Figure 27 shows the eclipse times predicted by the Antikythera mechanism compared with the true times at the geographical longitude of best fit. As can be seen, the AM does a reasonably good job of predicting the times of eclipses (RMS discrepancy of 2.52 h). However, the AM agrees somewhat better with the Babylonian-style theory of Sect. 19 (RMS discrepancy of 1.76 h) than it does with the actual eclipses.

Conclusion of Sect. 20: The geographical longitude of best fit to the eclipse times is 1.69 h (25.3°) east of Greenwich, roughly in the middle of the Greek cultural zone. But even at the longitude of best fit, the Antikythera mechanism agrees better with the Babylonian-style theory described in Sect. 19 than it does with the real eclipses.

21 Casting the net more broadly

Until now, we have assumed that the Saros dial eclipse pattern was inspired by Babylonian records, or, at least, was consistent with some Babylonian conventions. In this

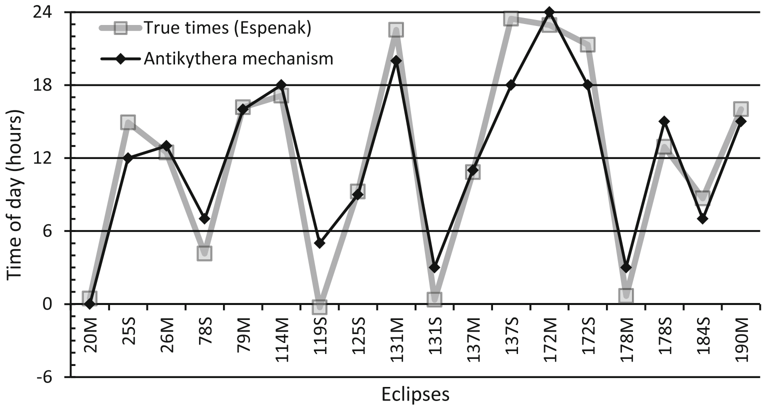


Fig. 27 True times of the eclipses on the geographical longitude of best fit compared with the times given by the Antikythera mechanism

section, we will show that we arrive at exactly the same result without assuming knowledge of the details of Babylonian eclipse theory. In "Appendix 3," we show that eclipse 13S is necessarily a D_1. We will use just this datum plus the result of Sect. 13, based on the lunar and solar anomaly analysis (which is independent of any assumption of Babylonian influence), to show that, nevertheless, our candidate of −204 is still the only one possible. This section, therefore, could be understood as an alternative to Sect. 12 for those readers who question the dependence of the Saros dial on Babylonian conventions.

Figure 28 is similar to Fig. 11, but now we assume that the eclipse pattern 8-8-7-8-7- could start from *any* D_1, not just a D_1 that is attested in Babylonian records as beginning a 8-8-7-8-7- or a D_1 that the Espenak catalog suggests could function well as the first eclipse of an 8-8-7-8-7- scheme. So, for many of these possibilities, the eclipse scheme would not function very well (meaning that it would generate many eclipse possibilities where there were no eclipses at all), but this is not problematic: It only shows that we are considering a much wider set of possibilities. From Fig. 28, it is clear that, besides solar Saros series 44, there is just one other series for which the lunar anomaly at the opposition of month 1 is close to 180°: solar Saros series 50, for years close to year 0.

Therefore, we would not be able to exclude series 50 on the basis of the lunar anomaly alone. Nevertheless, series 50 may be ruled out in a number of ways. Perhaps, the easiest way to see this is to make a total squared error over a Saros (TSEOS) graph for Saros series 50, which is shown in Fig. 29. We let each D_1 solar eclipse in Saros series 50 function (in turn) as a candidate for the eclipse of month 13. When we have assumed that a given D_1 eclipse is 13S, it is an easy matter, just by counting months, to identify each other eclipse on the AM Saros dial with a particular eclipse in the Espenak catalog. But once we choose a series 50 eclipse for month 13, we must still specify whether to take the times on the Antikythera mechanism as we find them, or to add 8 h to them, or to add 16, corresponding to the three possible positions of the exeligmos dial. Then, for each choice that we make (of the eclipse at month 13 and

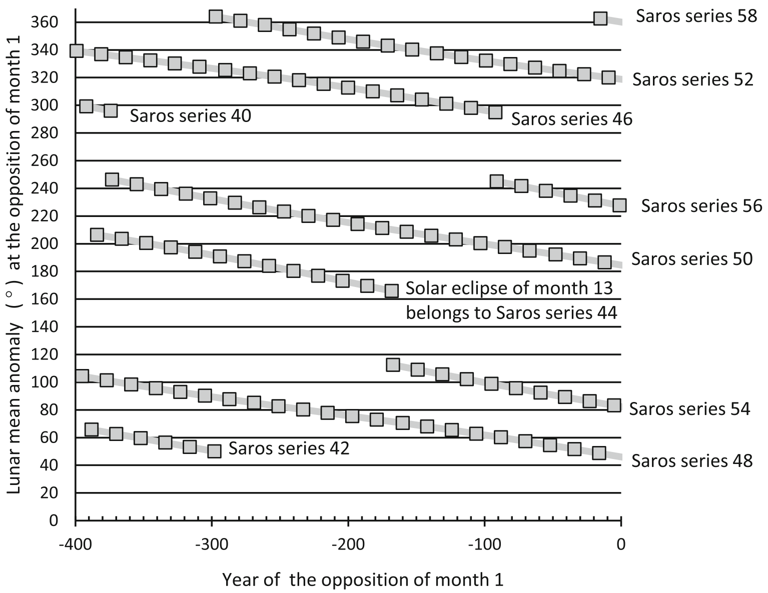


Fig. 28 Casting the net more broadly: No solar Saros series producing eclipses at the descending node gives a lunar mean anomaly near 180° at the opposition of month 1 in the period possible for the construction of the Antikythera mechanism, with the exception of series 44 and 50—and series 50 only for dates in the first century BCE

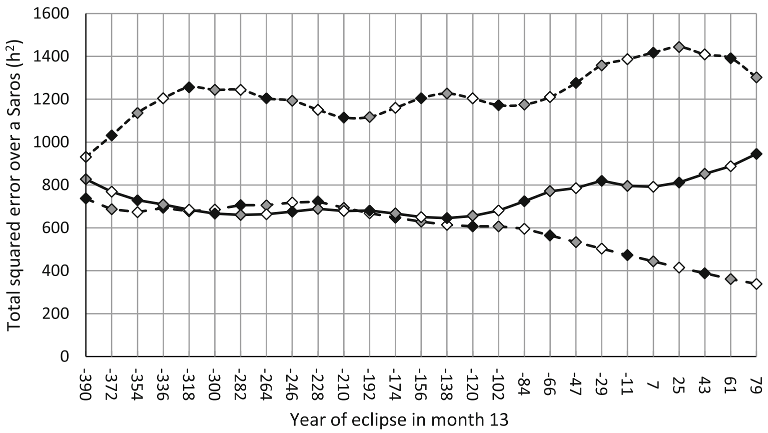


Fig. 29 A TSEOS graph for solar Saros series 50. *Black points* Nothing added to the AM times. *Gray* 8 h added to the AM times. *White* 16 h added to the AM times. 18 eclipses were used (four outliers excluded: 13S, 120M, 125M, and 184M)

of the position of the exeligmos dial), we calculate the total squared error over all 18 eclipses. To do this, we subtract the time of day of the actual eclipse (converted to local Aegean time) from the glyph time and square the difference. These differences are added up for all 18 eclipses. The result is plotted as a single point in Fig. 29. For each predicted and actually occurring eclipse, the glyph time was compared with the time of greatest eclipse, in UT. If an eclipse did not actually occur, the glyph time was compared with the time of true conjunction or true opposition, expressed in UT.

As can be seen, we obtain three curves. For example, if we suppose that the eclipse of −66 fell in month 13, we can either add 0 h to all the AM times (black point), add 8 h (gray) or add 16 h (white). These three points for −66 lie on three different curves. Points lying on a single curve are all consistent with one another as far as the calibration of the exeligmos dial goes, but assume different eclipses for month 13. The TSEOS graphs for series 50 show no well-defined minimum at all, and the total error is much higher than for Saros series 44. No calibration of the exeligmos dial is strongly preferred. Saros series 50 is rejected. (In "Appendix 7," we also prove that Saros 50 is inconsistent with the requirements imposed by the solar and lunar equations on the time of year and time of day of the opposition of month 1).

But now let us *cast the net more widely yet*: We abandon also the requirement that the lunar anomaly should be near 180° at the opposition of month 1. In Fig. 30, we plot TSEOS graphs for all extant solar Saros series producing eclipses at the descending node. For each D_1 that occurs in the Espenak catalog, we give that eclipse an opportunity to compete for the role of eclipse 13S. Then, we compute the total squared error in the eclipse times, summed over all 18 (excluding the outliers) eclipses on the dial. In constructing the graph, we use for the "actual time" of the eclipse the time of true conjunction or true opposition. (This has the advantage that we may treat uniformly eclipses that really occurred and eclipses that were predicted but did not occur). And we use, as before, the longitude correction of $1^{h}35^{m}$ to pass from UT to "local Antikythera time."

For each D_1 eclipse that is a candidate to be 13S, we of course have to do this three times, because in each run, we must choose whether to use all eighteen glyph times just as they come from the AM, or to add 8 h to all of them, or to add 16 h to all of them. In Fig. 30, solid black points indicate that the AM times are used directly, with nothing added to them. Gray points indicate that 8 h have been added to all the AM times. Hollow white points indicate that 16 h have been added to all the times.

In Fig. 30, it will again be seen that each solar Saros series gives rise to three graphs, corresponding to the three possible calibrations of the exeligmos dial. For example, Saros series 44 has one very low-lying graph. There is also a high-lying graph, and a graph that starts out at mid-level but rises steeply. In all three of the series 44 graphs, each eclipse in the horizontal row "Series 44" of Table 10 gets its chance to compete for the role of 13S. Thus, each single point on a series 44 curve of Fig. 30 corresponds to selecting one of the possible eclipses from the series 44 row of Table 10 to be 13S. It will be seen that for the eclipse of −203 May, on the lowest lying series 44 curve, the corresponding point is black. On the high-lying series 44 curve, the −203 point is gray. And on the rising curve, the same point is white. Starting from a given date, on one curve the points run black, gray, white, …; on another they run gray, white, black, …; and on the third they run white, black, gray, …. Thus, in this one figure, we have allowed all D_1 eclipses (from any of ten extant Saros series) to compete for the role of 13S and, for each of these cases, we have considered all three possible positions of the exeligmos dial. Solar Saros series 44 stands out as

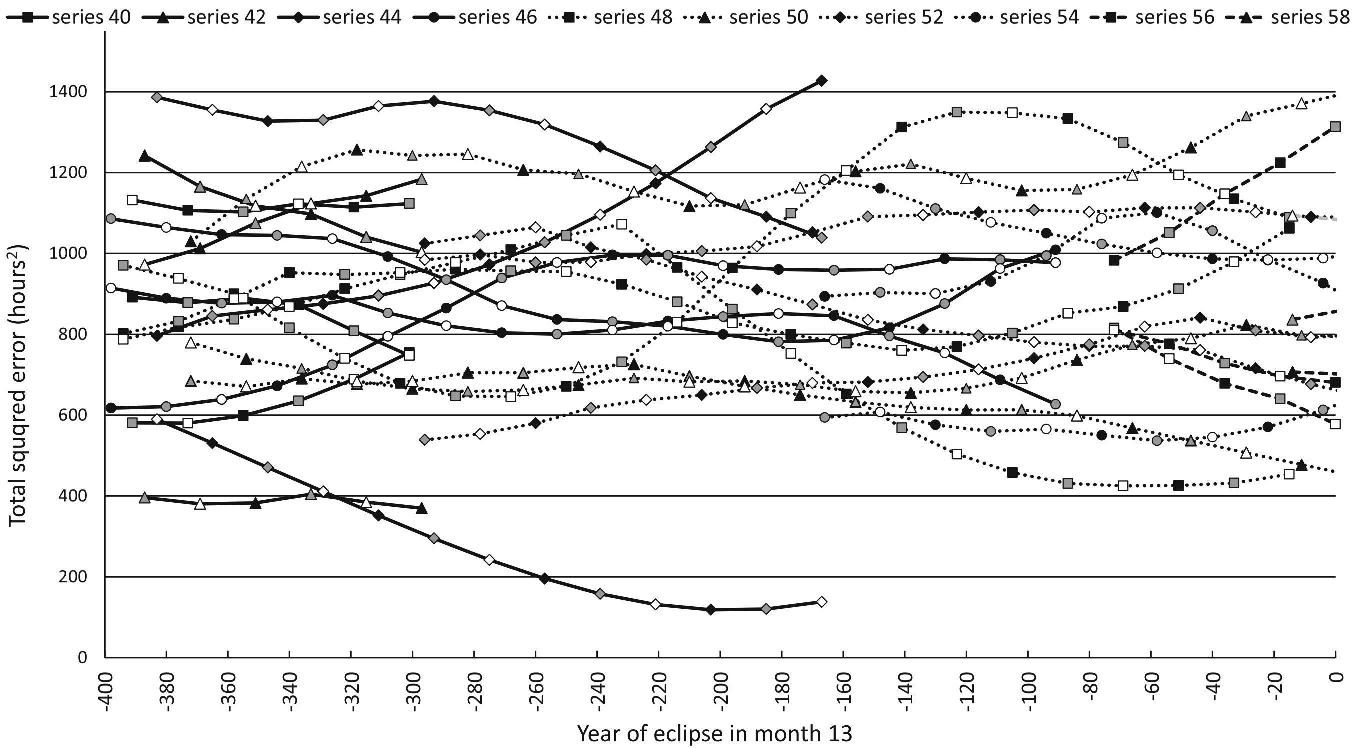


Fig. 30 TSEOS graph for ten solar Saros series. *Black points* nothing added to the AM times. *Gray points* 8 h added to the AM times. *White points* 16 h added to the AM times. Note that there are three curves for each Saros series, reflecting the three possible calibrations of the exeligmos dial. A single solution stands out: The solar eclipse of month 13 belongs to solar Saros series 44, with 0 added to the eclipse times for the Saros cycle starting in −203, 8 h added to the times for the cycle starting in −185, 16 h added to the times for the cycle starting in −167, and so on. A curve ends if its Saros series ends. A curve begins when a new Saros series moves into the D_1 position

unique. No other choice gives such a low-lying error curve, with a deeply defined minimum.

Conclusion of Sect. 21: Even if we discard the assumption that the solar eclipse of month 13 was drawn from the boldfaced candidates in Table 12 and cast the net much more broadly, all solar Saros series except 44 can be eliminated. Consequently, we are left with the unique date of −203 May 17 for eclipse 13S.

22 Some confirming evidence

22.1 Date of the opposition of month 1 deduced from the solar anomaly

In establishing the epoch of the Saros dial, we used a sort of sieve of Eratosthenes to systematically remove possibilities, until we were left with a single date. In this approach, the lunar mean anomaly played a key role, as it allowed us to rule out all but solar Saros series 44 for the eclipse of month 13. But we did not need or make any use of the solar equation of center. The solar fit, however, provides important confirming evidence.

If we take the longitude of the Sun's apogee to be about 65.5°, which was Hipparchus's value, and which, more importantly, was about right for the era of the Antikythera mechanism, the mean longitude of the Sun at the opposition of month 1 would be in the range 35°–66°.[24] These longitudes correspond to dates of roughly April 26 to May 27 (Julian calendar). That is, according to the result of fitting the solar equation of center, the full Moon of month 1 should have been between late April and late May. This is in good agreement with the result of matching the absolute times of eclipses, which gave us the full Moon of −204 May 12 as the best fit for month 1. Thus, the surviving material is redundant—there is more than the minimum necessary for establishing an epoch. It is important that two different approaches lead to concordant results.

In looking at the TSEOS graphs of Figs. 17 or 18, one can see that −203 is favored for the solar eclipse of month 13, but are −257 and −149 so far from the bottom of the well that they must be excluded? Adopting either of these dates would give trouble with the requirement that opposition of month 1 lies between April 26 and May 27. For example, if we pick the eclipse of −149 June 19 for month 13, the full Moon of month 1 would be that of 150 June 14, which is too late in the year. Similarly, if we pick the eclipse of −257 April 15 for month 13, then the full Moon of month 1 would be that of −258 April 10, too early in the year. Thus, the interlocking requirements of the TSEOS graphs and the solar anomaly analysis provide a tightly constrained solution.

[24] Recall that $\bar{\lambda} = \bar{\alpha} + \Pi$, where for the Sun we may put $\Pi = 65.5° - 180°$. Thus, $\bar{\lambda} = \bar{\alpha} - 114.5°$. For the arbitrary value of the solar mean anomaly $\bar{\alpha}$, put in any of the initial values of the mean solar anomaly $\varphi_{\odot}$ from column 5 of Table 12. For example, if we use $\varphi_{\odot} = 175.5°$, we find $\bar{\lambda} = 61.0°$ for the mean longitude of the Sun at the opposition of month 1.

22.2 Value of $\bar{h}_0$

As mentioned in Sect. 13, the parameter $\bar{h}_0$, found by fitting the lunar and solar anomalies, represents the time of day of the opposition of month 1. Our result was **between 14:35 and 16:02**, in the local time appropriate to the Antikythera mechanism. We also found that the opposition of month 1 best corresponds to that of −204 May 12.

Consulting Espenak (Six Millennium Catalog of Phases of the Moon) on the NASA Eclipse Web Site, we find that the full Moon of −204 May 12 fell at 13:21 UT. For the meridian appropriate to the Antikythera mechanism, we use 25.3° E, the longitude of best fit from Sect. 20. (Here are some representative longitudes that help define the Greek cultural zone: Alexandria 29.9°E, Athens, 23.7°, Syracuse 15.3°E). Converting from UT to "Antikythera mechanism time," we find that the full Moon of −204 May 12 fell at $13{:}21 + (25.3/15)^{\mathrm{h}} = \mathbf{15{:}02}$. This would be the local time for 25.3°E longitude. We could add 18^{m} for Alexandria, or subtract 40^{m} for Syracuse and still be very close to the required zone. In short, the required time of the opposition of month 1 is a very good match to the opposition of −204 May 12.

22.3 The fiducial mark[25]

Most of the moving parts of the mechanism were actuated by gears driven by a single input. However, one part had to be moved by hand. This is the Egyptian calendar ring, which was divided into the 12 months (30 days each) and five additional days of the Egyptian year. Because the Egyptian calendar year was always 365 days long, with no leap days, the calendar ring had to be displaced "by hand" by one day every 4 years. Beneath the Egyptian calendar ring is a circle of closely spaced holes drilled into the underlying plate. There was probably a little post (or posts) on the back of the calendar ring. The ring could therefore be pulled off, turned to the appropriate orientation for the year under consideration, and then plugged back in.

On the plate just outside the Egyptian calendar scale, near the beginning of the month of Payni, is an apparently engraved radial mark. Price (1974, pp. 19–20) argued that this was a fiducial mark for setting the Egyptian calendar ring for some initial date. But in his analysis Price assumed that the calendar ring is still in its original position and, when this led to impossible dates, that it was set at the correct day of the month, but the wrong month of the year. However, as is known, the Egyptian calendar ring is out of its proper position by several months for the epoch of the Antikythera mechanism, so no inference can be drawn from the day of the year that now happens to lie against the fiducial mark. But, since the mark is inscribed on the same plate as the zodiac, something interesting can be said about the zodiac degree corresponding to the mark. To be sure, the X-ray computed tomography (CT) scans show that the plate in the vicinity of the mark is badly cracked, so one could wonder whether this is a deliberately made mark or some sort of damage.

We point out that the fiducial mark is almost perfectly radial. (See Evans and Carman (2014), Figure 1 on p. 155). The radial direction of the mark supports the

[25] The first parts of this section are based on material in Evans and Carman (2014).

view that it is indeed associated with the scales. Like Price, we ask whether it might have been intended as the "$t = 0$" setting mark for the Egyptian calendar ring. Of course, it could be conceivable to prescribe the setting of the calendar ring without the use of a separate fiducial mark, if, for example, there were an inscription that said: for such and such a year, place 1 Thoth against a certain degree of the zodiac. We are lucky in the portion of the zodiac that is preserved, amounting to approximately a quadrant. For the Sun's position on the first of Thoth fell in the extant portion of the zodiac between the years −424 and −71. This encompasses practically the whole range of possible dates for the construction of the Antikythera mechanism, except perhaps for a very few years at the most recent end of the interval, immediately before the shipwreck. Thus, if there *were* a calibration mark for the first of Thoth, it would almost certainly have to fall in the preserved portion of the zodiac. There is one, and only one, such mark visible in the CT and, as there is only one, it is likely the setting mark for the calendar scale. But we acknowledge that in the research community, opinion is divided about whether this mark is intentional or accidental.

Let us enquire for just which year the beginning of Thoth would be aligned with the fiducial mark. In the list below, for each year in column 1, column 2 indicates the Julian calendar date corresponding to 1 Thoth, taken from Bickerman (1980, pp. 115–112). Column 3 gives the longitude of the Sun calculated from modern theory for noon of 1 Thoth in the given year, at 23° East longitude. For ΔT we used $3\frac{1}{2}^{\mathrm{h}}$, which is appropriate for the years around −200.

The fiducial mark lies at about Libra 17.7°, i.e., longitude 197.7°, according to the modern convention, which assigns to the first mark of Libra the value 0°. However, it is probable that the ancient mechanic considered the long mark at the beginning of Libra to represent Libra 1°, which means the fiducial mark corresponds to 198.7°. To allow for either possibility, we look for years in which the Sun's noon longitude on the first of Thoth falls in the range from about 197.2°–199.2° (thus allowing half a day one way or another about noon for either possibility). As can be seen, the result is the range −213 to −205 (bold print in the table). But we do not know just how the ancient mechanic would have calculated solar longitudes for this calendrical problem. Would he simply have used mean longitudes, for example? Moreover, how accurate was the equinox or solstice date that was used to tie the Sun to the calendar? If we allow a total of $2\frac{1}{2}°$ (roughly the size of the maximum solar equation) above 198.7° and below 197.7°, we look for years for which the Sun's longitude at local noon on the first of Thoth fell in the broader range 195.2°−201.2°. This gives us the more conservative estimate of −221 to −197. That the fiducial mark gives a range of years that includes our likely epoch for the Saros dial may be taken as supporting evidence—but only, of course, if one believes in the interpretation of this mark that we have sketched.

Longitudes of the Sun (calculated from modern theory) at noon on the first day of Thoth, for geographical longitude 23°E

Year	1 Thoth	Sun
−197	Oct. 12	195.2
−201	Oct. 13	196.2
−205	**Oct. 14**	**197.2**
−209	**Oct. 15**	**198.2**
−213	**Oct. 16**	**199.1**
−217	Oct. 17	200.1
−221	Oct. 18	201.1

Now, as we have seen, the middle of month 1 of the Saros dial fell in the month of May. But the first month of the Metonic dial is Phoinikaios, the first month of the civil year in the family of calendars related to the calendar of Corinth. Strong evidence requires this month to fall in August or September (Paul Iversen and John Morgan, personal communication; Iversen 2013; Morgan 2013). Thus, it is clear that the first month of the Saros dial and the first month of the Metonic dial do not correspond to the same month. This was a surprise to us, and at first a disappointment, as we had hypothesized that the two dials would begin at "month 1" together. This also would have had great simplifying advantages in the search for an epoch. But in fact the two pointers do not start off together, with each pointing to the first month of its spiral on "day 1." As we have seen, there is a simple and plausible explanation for the choice guiding the structure of the Saros dial: At the full Moon of month 1, both the Moon and the Sun were at apogee. The first cell of the Metonic dial is, naturally enough, the first month of the civil year.

Now, does the fiducial mark correspond to the epoch of the Saros dial or to the epoch of the Metonic dial? The fiducial mark connects the (approximately) solar year of the Egyptians with the Sun's motion around the zodiac. Since the Greek luni-solar year is also tied to the zodiac (with some sloshing back-and-forth in accordance with the 19-year Metonic cycle), the fiducial mark can be understood as providing a statement about calendars. In a way it provides the link between the Egyptian and the Greek calendar, through the intermediary of the zodiac. There is no reason why it should have anything to do with the Saros cycle directly.

Thus, we believe that the fiducial mark indicated the zodiac position of the first of Thoth in year 1 of a certain Metonic cycle. In this paper, we have given strong evidence for putting the first month of the Saros dial in −204. The fiducial mark (if such it is) suggests a Metonic cycle that began between −221 and −197. Of course, there is no reason for there to be a direct connection between a Metonic and a Saros cycle. They run with their own periods (19 years versus about 18 years and 11 days). And if one is constrained to start at the beginning of the civil year and the other to start when the Sun and Moon are both at apogee, it would require nearly a miracle to have them start out together.

Now, there were two plausible choices for the reckoning of years in terms of Metonic cycles. The ancient mechanic might conceivably have reckoned from −431, the year of Meton and Euctemon's famous summer solstice. Or he might have reckoned from

−329, the first year of the first Callippic cycle, a reform and extension of what Meton had started. For reckoning by Meton, new 19-year cycles would begin in −222 and −203 (these are the only choices within, or nearly within, the limits −221 to −197). For reckoning by Callippus, new 19-year cycles would begin in −215 and −196. The Callippic convention should probably be preferred here, in view of the fact that an inscription on the mechanism mentions the 76-year (Callippic) period, and it is believed that a subsidiary dial on the mechanism indicated the place of the current 19-year cycle within the 76-year period (Freeth et al. 2006). −215 was, of course, within the lifetime of Archimedes, which will either tantalize one with possibilities or repel one as wildly improbable. −196 is slightly outside our range of likely interpretations of the fiducial mark, but not so far outside as to justify rejection.

Conclusion of Sect. 22: Three pieces of supporting evidence are offered. (1) The fit to the solar equation of center implies that the opposition of month 1 fell between April 26 and May 27 (Julian calendar), which is in accord with the date determined from fitting the absolute times of the eclipses. (2) The parameter $\bar{h}_0$ *implies that the opposition of month 1 occurred between 14:35 and 16:02 local time. This is a good fit to the actual time of the opposition of −204 May 12. (3) The fiducial mark (if it is genuine) may indicate the position of the first of Thoth for the first year of a Metonic cycle that began between -221 and −197.*

23 Summary and closing discussion

If we assume that the solar and lunar eclipses were placed on the Saros dial in conformity with Babylonian 8-8-7-8-7- patterns, then 14 different reconstructions are consistent with the extant eclipse glyphs. If we invoke the Babylonian convention that the 8-8- of a solar Saros scheme should start at the descending node and that the 8-8- of a lunar Saros scheme should start with the Moon at the ascending node, and if we make use of the apparent fact that the southernmost solar eclipses of each diagonal sequence were omitted, these 14 combinations are reduced to a single solution, which we call 7β.

The eclipse predictor of the Antikythera mechanism works best if the full Moon of month 1 of the Saros dial corresponds to −204 May 12. This we shall refer to as the likely epoch of the eclipse predictor. Further, the exeligmos dial should read zero for the Saros cycle starting on −204 May 12.

The solar eclipse of month 13 belongs to solar Saros series 44 (this particular result is very strong, the strongest result of this investigation), and the eclipse predictor will work best if this is the eclipse of −203 May 17.

At the epoch, both the solar anomaly and the lunar anomaly were close to zero measured in the ancient way from apogee (or close to 180° if measured from perigee in the modern way). The lunar anomaly alone is enough to secure the dating. But the solar equation also implies that the first cell of the Saros dial corresponded roughly to May, which provides confirmation for the dating by means of the lunar anomaly.

Calendrical evidence implies that the first month of the Metonic dial probably corresponds to August or September. Thus, the first cell of the Metonic dial and the first cell of the Saros dial do not represent the same month.

The relation between the lunar and solar eclipses follows the alternative Steele rule. This suggests that the adaptor drew on a Babylonian solar eclipse list that had already moved over to a new calibration, but a lunar eclipse list that was still running on the previous calibration.

In Sect. 21, we were able to find a likely epoch date without making use of Table 10 (the list of eclipses historically used by the Babylonians to start a solar 8-8-). The epoch nevertheless agrees with the epoch we found using the Babylonian record. Some extra robustness is therefore lent to the epoch date determined.

It is plausible and perhaps likely, but not possible to prove statistically, that the corrections to the mean times of the eclipses were done on the basis of Babylonian-style "equations of center"—a quadratic form for the Moon and a piecewise-linear form for the Sun. Given the likely epoch of the eclipse predictor, it is reasonable to exclude a trigonometric equation of center, since the likely epoch is before the development of trigonometry. In any case, a method of prediction based on Babylonian-style equations of center and the associated daily velocities reproduces the eclipse times of the AM rather well. The eclipse times on the AM agree better with the results of this prediction scheme than they agree with the real times of the eclipses.

The geographical longitude of best fit (obtained by matching the AM eclipse times to the actual times of the eclipses) corresponds to the Aegean Sea with an uncertainty of about an hour of longitude.

What was the likely date of fabrication? The eclipse predictor works best for the years −204 to −186. It could run for a couple of Saros cycles before or after that, using the 8- and 16-h corrections from the exeligmos wheel, but the eclipse times would be less accurate. Was the machine fabricated in advance of −204, with the idea that it would be at its prime from −204 to −186? This would suggest a fabrication date somewhat before −204. Or were eclipse data compiled during the Saros of −204 to −186 (which would perhaps explain why the eclipses best fit this period)? This would imply a construction somewhat after −186. And, of course, we cannot exclude the possibility that a Greek mechanic used an old and outdated eclipse list, hoping that the 8-h rule would keep it relevant to his own day.

While it is not possible to be certain, there are pieces of evidence that bear on the issue. The solar eclipse predictor follows one calibration of the Saros scheme, but the lunar eclipse predictor was still running on an older calibration. This suggests a fabrication date reasonably close to a Babylonian recalibration of the Saros scheme. The approximate dates of these recalibrations are shown in Table 10—thus sometime around −261, around −203, or around −109. But the timing errors of the eclipses would be significantly larger around −261, and very much larger around −109. Again, −203 looks like the best candidate.

Also, it would not have been possible to construct an entire sequence of solar and lunar eclipse times based on observations over one Saros cycle (from −204 to −184, say), as too few of the eclipses would be visible from any one place; so this suggests prediction form theory, which means that there is no strong reason to prefer a date after −184. Moreover, for the Babylonians, *observed* times were frequently expressed in terms of the interval between the beginning of the eclipse and sunrise or sunset (Steele 2000b, pp. 57–58), often measured in terms of the *uš* (or "time degree," which is one 360th of the day and night). And, according to Steele (2000b, p. 66), a typical Babylonian eclipse timing, running, e.g., for about 2^h40^m has an accuracy of about $\pm 24^m$, which is quite a bit better than the typical timing errors for predicted eclipses. Thus, we can see that Babylonian records of observed eclipse times would not have been in a form that could have been easily or directly used. And, if the eclipse times of the AM were based directly on observations, the timing errors should be smaller than they are.

By contrast, as we have seen, the errors in the eclipse times are of about the right size to be consistent with prediction. Thus, while we cannot be certain, a plausible inference is that the eclipse times were predicted to run for the Saros cycle −204 to −184, which suggests a fabrication time close to −204, and possibly somewhat before. Finally, the fiducial mark suggests the mechanism was keyed to a Metonic cycle that began within about ±12 years of −209.

There are more data extant on the Saros dial than the minimum needed to establish a likely epoch; the resulting crosschecks make the epoch date reasonably strong. Because the conclusions drawn in this paper are so tightly constrained, it would be easy to check them and to refute or refine them if more fragments of the Saros dial should ever come to light.

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24 Appendix 1: The possible distributions of lunar eclipses on the Saros dial

In their study of the lunar eclipses, Freeth et al. rightly remark that the critical factor in the analysis is where the 5-month intervals are. So, in their analysis, they fix one of the "7-" groups, identifying two 5-month intervals, one between months 79 and 84 and the other between 120 and 125. Then, they say,

> Therefore any Babylonian scheme that is consistent with the glyphs must have 5-month [intervals] between Months 79 and 84 and between months 120 and 125. This fixes one of the '7's in the 8-7-8-7-8 pattern and it is easy to see that this then fixes the whole Babylonian pattern relative to the glyphs (Freeth et al. 2008, p. 32).

But it is not correct to suppose that knowing one of the "7-" groups is enough for fixing the whole pattern. In the Babylonian scheme, there are two different "7-" groups—one followed by two groups of 8 eclipses and the other followed by only one—so it is necessary to know which of the two "7-" groups has been identified. Thus, it turns out that there is not just one possible distribution of lunar eclipses that is consistent with the Babylonian scheme and with the inscriptions on the Antikythera mechanism, but rather two different distributions, which are in Table 1.

25 Appendix 2: The possible distributions of solar eclipses

Solar EPs are inscribed in months 13, 25, 72, 78, 119, 125, 131, 137, 172, 178, and 184. As Freeth et al. noted, between cell 184 and cell 13 (carrying forward beyond the end of the 223-month dial into the beginning of the next Saros cycle), there must be two 4-month gaps. For between cell 184 and cell 13 are cells for 51 months, which means that we have room for an "8," plus 2 4-month gaps (one before and one after the "8"). Alternatively, we could have a "7", plus 2 4-month gaps (one before and one after the "7"), plus six extra months that would belong to the next or the previous group. Thus, the 2 4-month gaps could be separated either by an "8" or by a "7".

Case i Suppose that the 2 4-month gaps are separated by an "8." Then, the 4-month gaps must be at the extremes, i.e., one just before month 13 and the other just after month 184. So, the 2 4-month gaps consist of months 9–12 and 185–188. And there is an "8" from 189 through 8. All three solutions under Case i satisfy these conditions.

The Saros pattern takes the form 8-8-7-8-7- (or a cyclic permutation thereof). So we will have sub-cases, depending on which of the three possible positions is taken by the "8" that occupies cells 189 through 8. We must allow for the possibility that this "8" is:

the second "8" of two successive "8"s (subcase 1)
or the first "8" of two successive "8"s (subcase 2)
or the lone "8" (subcase 3).

Suppose, alternatively, that the two 4-month gaps (between cell 184 and cell 13) are separated by a "7". Then, we have two additional cases:

Case ii One of the 4-month gaps is just after month 184, and is followed by the "7". Thus, there is a "7" from 189 through 2. Both solutions under Case ii satisfy this condition. This "7" is either:

the "7" that follows two successive "8"s (subcase 4)
or the "7" that follows a lone "8" (subcase 5).

Case iii One of the 4-month gaps is just before 13 and is preceded by the "7". Thus, there is a "7" from 195 through 8. Both solutions under Case iii satisfy this condition. This "7" is either:

the “7” that follows two successive “8”s (subcase 6)
or the “7” that follows a lone “8” (subcase 7).

So, to sum up, there are seven solar patterns that would be consistent with the extant glyphs and with a Babylonian-style 8-8-7-8-7- distribution scheme and these are shown in Table 2.

26 Appendix 3: Alternative determination of the A and D nodes for the solar eclipses

We mentioned above that in the Babylonian Saros record the first eclipse of an 8-8- group is consistently a D (an eclipse at the descending node). But there are no astronomical reasons for that. It is clearly a convention, because the first eclipse of an 8-8- group could just as well be an A. So, if we do not want simply to assume that the Saros dial of the Antikythera mechanism was based on the Babylonian records, we have to explore both possibilities. The sequence actually followed by Babylonian records, starting with a D eclipse at the beginning of the 8-8-, is represented in the left-most column of Table 9. The alternative sequence, starting with an A eclipse, can be obtained by exchanging A for D in the same column.

Now, we know that the solar eclipse in month 13 comes 11 months after a solar eclipse in month 2. (For all seven possible solar solutions require an eclipse in month 2). Only eclipses that are first or second in the 8- or 7- groups are 11 months after another EP. Thus, whether we start the 8-8- with a D or with an A, we will always have an A_1 and D_1 as the first two EP. (What will change is the order—either D_1 A_1 or A_1 D_1). Now, applying the omission rule, we know that A_1 eclipses must be omitted; but the eclipse in month 13 is an extant eclipse: Consequently, it is a D_1. Therefore, no matter which of the seven solar patterns were adopted for Saros dial, and no matter whether the 8-8- began with an A or a D, we can be sure that the eclipse in month 13 is a D_1.

27 Appendix 4: Examples of 24-h correction analysis applied to the inscribed eclipse times

For example, consider eclipses 78S and 119S (solar eclipses in months 78 and 119):

Eclipse	Change in lunar mean anomaly from eclipse 20M	Recorded advance in time from eclipse 20M	Advance in time from 20M expected due to mean motions	Advance in time from 20M over and above effect of mean motions
78S	251.03°	7^h	13.31^h	-6.31^h
119S	230.05	5	7.67	−2.67

By virtue of the number of anomalistic months elapsed from the opposition of month 20, we see that the Moon’s mean anomaly must have been similar in both eclipses—251° greater than the mean anomaly at the opposition of month 20 (in the case of eclipse 78S) and 230° greater than the mean anomaly at the opposition of month 20 (in the case of eclipse 119S). Thus, the effect of the lunar anomaly on the time shift between these two eclipses was small. The inscribed glyphs show that the time of day of eclipse 78S advanced by 7 h from the time of day recorded for eclipse 20M (the lunar eclipse of month 20). The number of synodic months elapsed from 20M implies that the time of day for eclipse 78S should have shifted forward by 13.31^h due to the mean motions alone. Thus, the advance in time over and above the effect of the mean motions was $7 - 13.31 = -6.31^h$, for eclipse 78S. Similarly, the advance in time of eclipse 119S with respect to 20M, over and above the effect of the mean motions was -2.67^h. The "signal" that we are trying to explain consists of the numbers in column (5)—the advance in time over and above the effect of the mean motions. Now, we have chosen two eclipses at which the lunar mean anomalies were similar; thus, the effect of the lunar anomaly on the signal is small. The two values in column (5) should rarely differ by more than 8 h, which is the maximum effect of the solar anomaly. The difference between −6.31 and −2.67 is <8 h, so we may be confident that no correction is required to the times in column (3).

But now consider eclipses 25S, 137S, 172M and 178S:

Eclipse	Change in lunar mean anomaly from eclipse 20M	Recorded advance in time from eclipse 20M	Advance in time from 20M expected due to mean motions	Advance in time from 20M over and above effect of mean motions
25S	322.06°	12^h	10.07^h	1.93^h
137S	334.98	18	21.00	−3.00
172M	326.10	24 (or 0)	16.54	7.46 (or −16.54)
178S	313.99	15	15.36	−0.36

These have lunar mean anomalies that span a narrow range of about 21°. So it is clear that the difference in the glyph times between 20M and 172M must be treated as 24 h and not 0 h. If we took it as 0 h, then the values in column (5) for 137S and 172M would differ by over 13½ h, which is out of the realm of possibility. Now, with column (3) for 172M chosen to be 24 h, it is true that the values in column (5) for 137S and 172M differ by 10½ h, a bit more than the 8-h maximum effect for the solar anomaly. But the lunar anomalies for these two eclipses differ by about 9°, so the maximum possible lunar effect on the time shift is about $10^h \sin(9°) = 1\frac{1}{2}$ h; thus a total lunar plus solar effect of up to 9½ h is conceivable, which is perhaps close enough to the attested 10½ h difference. For, we should certainly allow for the possibility of some scattered error (which, in view of the uncertainties about the time conversion, discussed above, could easily amount to an hour or somewhat more). Thus, all four eclipses are in decent accord, but only if we put column (3) at 24 h for 172M.

Consider one last example involving the eclipses 137M and 178M:

Eclipse	Change in lunar mean anomaly from eclipse 20M	Recorded advance in time from eclipse 20M	Advance in time from 20M expected due to mean motions	Advance in time from 20M over and above effect of mean motions
137M	142.06°	11^h (or −13)	2.63^h	8.37^h (or −15.63)
178M	121.08°	3	20.99	−17.99

Using the recorded advances in time with respect to 20M (11^h for the first eclipse and 3^h for the second) gives values in column (5) that differ by some 14.38 h. This far exceeds the 8-h maximum for the solar effect. Even when we add in a maximum possible lunar effect of $10^h \sin(21°) = 3.58^h$, for the lunar anomalistic difference of 21°, we could only hope to explain a difference of 11½ h in column (5). Thus, it is clear that either 24 h needs to be subtracted for 137M in column (3), or 24 h added for 178M. Deciding which is required involves fitting these two eclipses into the larger pattern.

28 Appendix 5: Identification of outliers

We suspect that, for several of the month cells, there is something wrong with the times of the eclipses inscribed on the Saros dial, perhaps a copying error or a calculation error.

Figure 31 displays the time by which each of 18 eclipses arrived early or late, by comparison with what would be expected from mean motions alone. The time is reckoned in synodic months from the opposition of month 1. So the basic signal we are trying to explain has an amplitude of some 10–15 h.

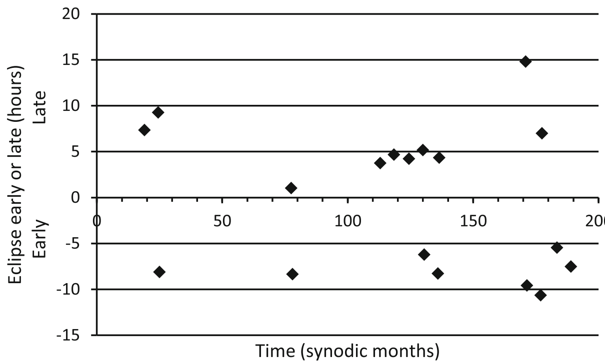

Fig. 31 Raw signal of the eclipse times, to be explained by lunar and solar equations of center. 18 eclipses are used (excluding 13S, 120M, 125M, and 184M). Plotted on the vertical axis is the time by which an eclipse came early or late by comparison with the time that would be expected from the mean motions acting alone. Plotted on the horizontal axis is the time elapsed since the opposition of month 1, measured in synodic months

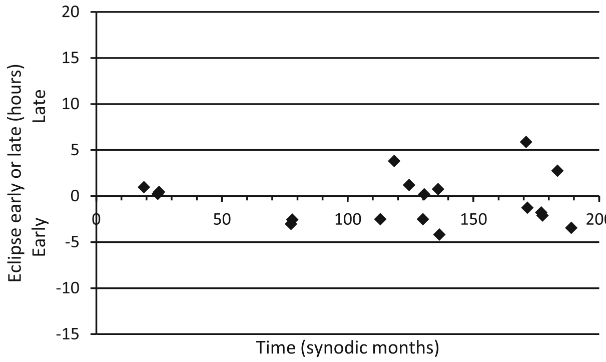

Fig. 32 Residuals in the eclipse times, once the lunar anomaly function is subtracted. The quantity plotted on the *horizontal axis* is the *vertical* value from Fig. 31 minus the lunar equation of center (with $A_{☾}$ and $\varphi_{☾}$ determined by the minimization of Q with respect to its five parameters)

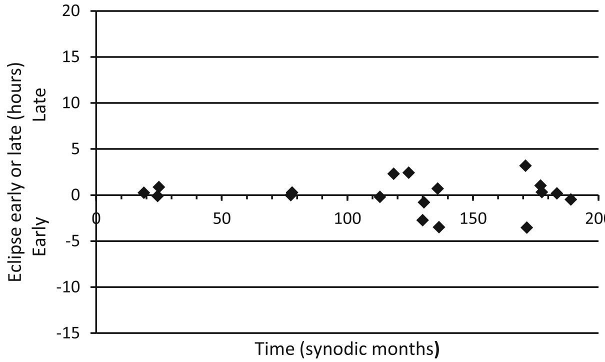

Fig. 33 Residuals, once both the lunar and solar anomaly functions are subtracted

In Fig. 32, we have plotted the data of Fig. 31 minus the lunar equation of center function. The graph therefore displays the residuals left over after the lunar equation of center is taken into account. The tightening of the plot shows overwhelming evidence of the existence of a lunar equation of center function in the basic data.

In Fig. 33, we plot the data of Fig. 31 minus the sum of the lunar and solar equations of center. The further tightening of the residuals (between Figs. 32, 33) is evidence of the existence of the solar equation of center in the basic data. Figure 34 is the same as Fig. 33, except that the four eclipses excluded as outliers (13S, 120M, 125M, and 184M) have been added to the figure and are represented by the hollow squares. (The solar and lunar fits are still determined by the eclipses represented by the 18 diamonds). Eclipses 13S and 125M are so out of character with the rest of the data, that their exclusion seems quite justifiable. For 120M and 184M, the case is less clear-cut. We exclude them for having residuals >5 h. The exclusion of these four eclipses produces a considerable improvement in the quality of the lunar fit (Fig. 7 contrasted with Fig. 9), which is a stronger argument for their validity than the improvement produced in the solar fit (Fig. 8 compared with Fig. 10). However, we present our results both with and without the exclusion of these outliers.

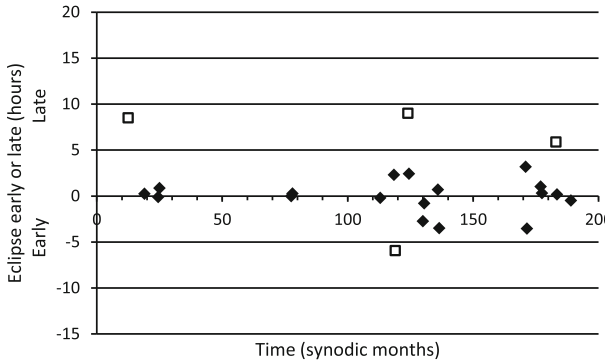

Fig. 34 Residuals in the eclipse times. The black diamonds are as in Fig. 33 and represent the observed times *minus* the contributions of both the lunar and the solar equations of center. The lunar and solar equations have been fitted using 18 eclipses represented by the *solid black diamonds*. The hollow *black squares* show the data points for the excluded eclipses 13S, 120M, 125M, and 184M

Alexander Jones (personal communication) independently found that the times of eclipses 13S, 125M, and 184M are problematical. The identification of outliers depends to some extent on the method used. He also finds that 172M is problematical (while we found 120M). In Fig. 14, 172M is the point with largest positive value and seems unproblematical; and in Fig. 33, it is the point with the greatest positive value, but does not really seem to be an outlier. A key point of technique is that it is important to fit the solar and lunar equations of center simultaneously. One will not get the correct fits by fitting the lunar equation first and then fitting the solar equation to the residuals, since these two functions are not orthogonal.

29 Appendix 6: The Babylonian node conventions and the Britton and Steele rules

As we have seen, the Babylonians put the first solar EP of an 8-8- consistently at the descending node D. And, after about −250, they put the lunar EP that starts the lunar 8-8- at the A node. We shall prove that this necessarily entails the Steele rule or the alternative Steele rule. On the other hand, if we keep the solar EP that starts the solar 8-8-7-8-7- at D, but also choose to place the lunar EP that starts the lunar 8-8-7-8-7- at D, this necessarily entails the Britton rule. Thus, the node conventions discussed in Sect. 6 are intimately linked with the Steele and Britton rules discussed in Sect. 5.

Let us reckon the solar eclipse pattern from the first eclipse in the solar 8-8-, at a moment we shall call time 0. And we shall measure time in synodic months (sm). The solar eclipse at the beginning of the 8-8- will of course be the most northerly D_1. We shall then prove that (a) if we want to start the lunar pattern from the most southerly A_1, we must necessarily follow Steele rule (or the alternative Steele rule). But, (b) if we want instead to start the lunar pattern from the most northerly D_1, then we must necessarily follow the Britton rule.

Proof of (a). An A_1 lunar eclipse falls 23.5 synodic months earlier or later than a D_1 solar eclipse. To see this, consider the D_1 solar eclipse at time 0. Then 4 solar EPs later, at time 24.0 sm, there will be another solar EP. There is a possibility of a lunar EP two weeks before this, at time 23.5 sm; for 23.5 sm × (242 dm/223 sm) = 0.5022 dm, over and above complete draconic months, or very nearly 180° (actually about 180.79°). Therefore, the lunar EP would occur very nearly 180° from the initial solar D_1, i.e., with the mean Moon at nearly the same distance from the node, but south of the A node (rather than north of the D node). And this is an A_1—the most southerly lunar EP. But, of course, we could have instead chosen to place a lunar EP at time −23.5 sm. Thus, the lunar pattern follows the same 8-8-7-8-7- scheme as the solar pattern, but it starts 4 EP earlier or later.

Proof of (b). The Britton rule implies that the first eclipse of the lunar 8-8-7-8-7-pattern is 111.5 synodic months before the first (D_1) solar eclipse of the solar 8-8-7-8-7. Now, 111.5 sm are *exactly* 121 dm. Therefore, the Moon would be exactly at the same distance from the node. Consequently, it would be also the most northerly D_1 of the lunar eclipses. Because the Britton rule implies an interval of exactly half a Saros cycle (111.5 sm), it is the same to move forward or backward: Going the one way or the other, we will wind up at an eclipse belonging to the same Saros series. So, there is no alternative Britton rule.

30 Appendix 7: Solar Saros series 50

We can use the time of the opposition of month 1 (deduced from the lunar anomaly analysis) and the time of year of the same opposition (deduced from the solar anomaly analysis) for an alternative test of solar Saros 50 as a candidate. In Table 17, we list the dates and local times that would be possible for the opposition of month 1, assuming eclipse 13S belongs to solar Saros series 50. We have considered eclipses of series 50 over four *exeligmos* periods, two before and one after our best *exeligmos* candidate. According to our analysis of the solar equation of center, the opposition of month 1 should be between April 26 and May 27, but oppositions of month 1 for all cycles

Table 17 Data for the analysis of solar Saros series 50: Possible dates and times of the true opposition of month 1. None of the possibilities are consistent with the requirements of the solar and lunar equations that the date be in April or May and that the time of day be between 14:35 and 16:02

Date of true opposition	Local time
−193 Oct 6	12:05
−175 Oct 16	19:56
−157 Oct 28	03:53
−139 Nov 7	11:55
−121 Nov 18	20:00
−103 Nov 29	04:07
−85 Dec 10	**12:14**
−67 Dec 20	**20:19**
−48 Jan 1	**04:20**
−30 Jan 11	12:15
−12 Jan 22	20:04
+6 Feb 2	03:45

from −193 to +6 are between October and January. We would have to go back to −427 to find an opposition of month 1 on May 19 (but there the lunar mean anomaly would be around 256°) or go forward to year +150 to find an opposition of month 1 on April 29. Also, the deduced time of day for the opposition of month 1 is between 14:35 and 16:02, and there is no time in the table even close to those limits.

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